5 CHAPTER

ALGEBRA

EXERCISE

YEAR : 1999

1. If a * b = 2a – 3b +ab, then 3 *5 +5*3 is equal to:
(a) 22 (b) 24 (c) 26 (d) 28

2. If $p \times q = p + q + \frac{p}{q}$, the value of 8 × 2 is:
(a) 6 (b) 10 (c) 14 (d) 16

3. Two numbers x and y $(x > y)$ such that their sum is equal to three times their difference. Then value of $\frac{3xy}{2(x^2 - y^2)}$ will be;
(a) $\frac{2}{3}$ (b) 1
(c) $1\frac{1}{2}$ (d) $1\frac{1}{3}$

4. The value of $\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\left(1+\frac{1}{x+3}\right)$ is:
(a) $1+\frac{1}{x+4}$ (b) $x+4$
(c) $\frac{1}{x}$ (d) $\frac{x+4}{x}$

5. If $x = 7 - 4\sqrt{3}$, then the value of $\left(x+\frac{1}{x}\right)$ is:
(a) $3\sqrt{3}$ (b) $8\sqrt{3}$
(c) $14+8\sqrt{3}$ (d) 14

6. If $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$, then the ratio $\frac{a+b}{b+c}$ equal to:
(a) $\frac{20}{27}$ (b) $\frac{27}{20}$ (c) $\frac{6}{8}$ (d) $\frac{8}{6}$

YEAR : 2002

7. If a*b =2(a + b), then 5*2 is equal to:
(a) 3 (b) 10 (c) 14 (d) 20

8. If $\frac{2a+b}{a+4b} = 3$, then find the value of $\frac{a+b}{a+2b}$
(a) $\frac{5}{9}$ (b) $\frac{2}{7}$ (c) $\frac{10}{9}$ (d) $\frac{10}{7}$

9. If a * b = a + b + ab, then 3 * 4 –2 * 3 is equal to
(a) 6 (b) 8 (c) 10 (d) 12

10. If $x \otimes y = 3x + 2y$. Then $2 \otimes 3 + 3 \otimes 4$ is equal to
(a) 18 (b) 29 (c) 32 (d) 38

11. If a : b = 2 :3 and b : c = 4 : 5, find $a^2 : b^2$: bc
(a) 4 : 9 : 45 (b) 16 : 36 : 45
(c) 16 : 36 : 20 (d) 4 : 36 : 40

12. If A : B = $\frac{1}{2} : \frac{3}{8}$ B : C = $\frac{1}{3} : \frac{5}{9}$ and C : D = $\frac{5}{6} : \frac{3}{4}$ then find the ratio of A : B : C : D
(a) 6:4:8:10 (b) 6:8:9:10
(c) 8:6:10:9 (d) 4:6:8:10

13. If A : B : C = 2 : 3 : 4, then $\frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ is equal to
(a) 8 : 9 : 16 (d) 8 : 9 : 12
(c) 8 : 9 : 24 (d) 4 : 9 : 16

14. If A : B = 1 : 2 , B : C = 3 : 4 and C : D = 5 : 6, find D : C : B : A
(a) 6 : 5 : 4 : 2 (b) 6 : 3 : 2 : 1
(c) 6 : 4 : 2 : 1 (d) 48:40:30:15

15. If $\frac{2a-5b}{3a+6b} = \frac{4}{7}$, then a : b is equal to
(a) 21 : 36 (b) 2 : 59
(c) 59 : 2 (d) 36 : 21

YEAR : 2003

16. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$ then $\frac{a+b+c}{c}$ is equal to
(a) 0 (b) 1
(c) 2 (d) 3

17. If $\frac{144}{0.144} = \frac{14.4}{x}$, then the value of x is
(a) 144 (b) 14.4
(c) 1.44 (d) 0.0144

18. If $1 < x < 2$, then the value of $\sqrt{(x-1)^2} + \sqrt{(x-3)^2}$ is
(a) 1 (b) 2
(c) 3 (d) $2x - 4$

19. If a ⊕ b = (a × b) +b, then 5 ⊕ 7 equals to
(a) 12 (b) 35 (c) 42 (d) 50

20. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to
(a) 1.45 (b) 1.88
(c) 2.9 (d) 3.7

21. If 47.2506 = 4A + 7B + 2C + $\frac{5}{D}$ + 6E, then the value of 5A + 3B + 6C + D + 3E is
(a) 53.6003 (b) 53.603
(c) 153.6003 (d) 213.0003

22. If $x * y = x^2 + y^2 - xy$, then the value of 9 * 11 is
(a) 93 (b) 103
(c) 113 (d) 121

23. If $5^{5x+5} = 1$, then x equals

(a) 0 (b) – 1 (c) 1 (d) $-\frac{4}{5}$

24. If $3^{x+3} + 7 = 250$, then x is equal to
(a) 5 (b) 3 (c) 2 (d) 1

25. If $\frac{1}{4}\times\frac{2}{6}\times\frac{3}{8}\times\frac{4}{10}\times\frac{5}{12}\times.....\times\frac{31}{64}=\frac{1}{2^x}$ the value of x is
(a) 31 (b) 32 (c) 36 (d) 37

26. If $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}$ & $y=\frac{\sqrt{3}-1}{\sqrt{3}+1}$, then value of x^2+y^2 is :
(a) 14 (b) 13 (c) 15 (d) 10

27. If $x = 3 + 2\sqrt{2}$, then the value of $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$ is:

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $3\sqrt{3}$

28. If p = 999, then the value of $\sqrt[3]{p\left(p^2+3p+3\right)+1}$ is

(a) 1000 (b) 999
(c) 998 (d) 1002

29. If $\frac{a}{b}=\frac{7}{9},\frac{b}{c}=\frac{3}{5}$, then the value of $a:b:c$ is
(a) 7 : 9 :15 (d) 7 : 9 : 5
(c) 21 : 35 : 45 (d) 7 : 3 : 15

30. If x : y = 7 : 3 then the value of $\frac{xy+y^2}{x^2-y^2}$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{7}$ (d) $\frac{7}{3}$

31. If [p] means the greatest positive integer less than or equal to p, then $\left[-\frac{1}{4}\right]+\left[4-\frac{1}{4}\right]+[3]$ is equal to
(a) 4 (b) 5 (c) 6 (d) 7

YEAR : 2004

32. The value of $\frac{(243)^{\frac{n}{5}}.3^{2n+1}}{9^n.3^{n-1}}$ is

(a) 1 (b) 9 (c) 3 (d) 3^n

33. If $x = 0.5$ and y = 0.2 then value of $\sqrt{0.6}\times(3y)^x$ is equal to
(a) 1.0 (b) 0.5 (c) 0.6 (d) 1.1

34. If $x^{x\sqrt{x}}=\left(x\sqrt{x}\right)^x$, then x equals

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{9}{4}$ (d) $\frac{3}{2}$

35. If $a = 7$, $b = 5$ and $c = 3$, then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ is
(a) 12 (b) –12 (c) 0 (d) 8

36. If $7^x=\frac{1}{343}$, then the value of x is

(a) 3 (b) – 3 (c) $\frac{1}{3}$ (d) $\frac{1}{7}$

37. $\dfrac{\frac{1}{3}.\frac{1}{3}.\frac{1}{3}+\frac{1}{4}.\frac{1}{4}.\frac{1}{4}-3\frac{1}{3}.\frac{1}{4}.\frac{1}{5}+\frac{1}{5}.\frac{1}{5}.\frac{1}{5}}{\frac{1}{3}.\frac{1}{3}+\frac{1}{4}.\frac{1}{4}+\frac{1}{5}.\frac{1}{5}-\left(\frac{1}{3}.\frac{1}{4}+\frac{1}{4}.\frac{1}{5}+\frac{1}{5}.\frac{1}{3}\right)}$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{47}{60}$ (d) $\frac{49}{60}$

YEAR 2005

38. If $0.13 \times p^2 = 13$, then p is equal to
(a) 10 (b) 0.01
(c) 0.1 (d) 100

39. If $\frac{x}{y}=\frac{3}{4}$, the value of $\frac{6}{7}+\frac{y-x}{y+x}$ is:

(a) 1 (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $1\frac{3}{7}$

40. If $x = 7 - 4\sqrt{3}$, then $\sqrt{x}+\frac{1}{\sqrt{x}}$ is equal to:
(a) 1 (b) 2 (c) 3 (d) 4

41. If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ & $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

42. If $a = 4.36$, $b = 2.39$ and $c = 1.97$, then the value of $a^3 - b^3 - c^3 - 3abc$ is
(a) 3.94 (b) 2.39 (c) 0 (d) 1

43. If $\frac{3a+5b}{3a-5b}=5$, then a : b is equal to
(a) 2:1 (b) 2:3 (c) 1:3 (d) 5:2

44. If $p : q = r : s = t : u = 2 : 3$, then $(mp+nr+ot) : (mq+ns+ou)$ equals:
(a) 3:2 (b) 2:3 (c) 1:3 (d) 1:2

45. If $x : y = 3 : 4$, then $(7x + 3y) : (7x - 3y)$ is equal to :
(a) 5 : 2 (b) 4 : 3
(c) 11 : 3 (d) 37 : 19

YEAR : 2006

46. For what value(s) of a is $x+\frac{1}{4}\sqrt{x}+a^2$ a perfect square ?

(a) $\pm\frac{1}{18}$ (b) $\frac{1}{8}$ (c) $-\frac{1}{5}$ (d) $\frac{1}{4}$

47. If $a \neq b$, then which of the following statements is true?

(a) $\frac{a+b}{2}=\sqrt{ab}$ (b) $\frac{a+b}{2}<\sqrt{ab}$

(c) $\frac{a+b}{2}>\sqrt{ab}$ (d) All of the above

48. If x, y are two positive real number and $x^{1/3} = y^{1/4}$, then which of the following relations is true?
(a) $x^3 = y^4$ (b) $x^3 = y$
(c) $x = y^4$ (d) $x^{20} = y^{15}$

49. If $x=\frac{\sqrt{3}}{2}$, then $\frac{\sqrt{1+x}}{1+\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}$ is equal to

(a) 1 (b) $2/\sqrt{3}$

(c) $2-\sqrt{3}$ (d) 2

50. If for non-zero, x, $x^2 - 4x - 1 = 0$, the value of $x^2+\frac{1}{x^2}$ is

(a) 4 (b) 10 (c) 12 (d) 18

51. $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x^2+\frac{1}{x^2}+1\right)$ is equal to

(a) $x^6+\frac{1}{x^6}$ (b) $x^8+\frac{1}{x^8}$

(c) $x^8-\frac{1}{x^8}$ (d) $x^6-\frac{1}{x^6}$

YEAR : 2007

52. If $a^{2x+2} = 1$, where a is a positive real number other than 1, then x is equal to
(a) – 2 (b) – 1 (c) 0 (d) 1

53. If x is real then the minimum value of $(x^2 - x - 1)$ is

(a) $\frac{3}{4}$ (b) 0 (c) 1 (d) $-\frac{5}{4}$

54. If $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$, then the value of a is

(a) $\frac{11}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{4}{3}$ (d) $\frac{-4\sqrt{7}}{3}$

55. If $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$ then $c+\frac{1}{a}$ is equal to

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

56. If $x=\sqrt{3}+\sqrt{2}$, then the value of $\left(x^3+\frac{1}{x^3}\right)$ is

(a) $6\sqrt{3}$ (b) $12\sqrt{3}$
(c) $18\sqrt{3}$ (d) $24\sqrt{3}$

57. If $x + y = 7$, then the value of $x^3 + y^3 + 21xy$ is

(a) 243 (b) 143
(c) 343 (d) 443

58. If $\otimes$ is an operation such that $a \otimes b = 2a$ when $a > b$, $a + b$ when $a < b$, a^2 when $a = b$,

then $\left[\frac{(5\otimes 7)+(4\otimes 4)}{3(5\otimes 5)-(15\otimes 11)-3}\right]$ is equal to :

(a) $\frac{1}{3}$ (b) $\frac{14}{23}$ (c) $\frac{2}{3}$ (d) $\frac{14}{13}$

YEAR 2008

59. If $(125)^x = 3125$, then the value of x is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{5}{3}$ (d) $\frac{5}{7}$

60. If $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$, then x is equal to

(a) $\frac{25}{4}$ (b) 4 (c) 9 (d) 16

61. If $n+\frac{2}{3}n+\frac{1}{2}n+\frac{1}{7}n = 97$ then the value of n is

(a) 40 (b) 42 (c) 44 (d) 46

62. If $x = 3+\sqrt{8}$, then $x^2+\frac{1}{x^2}$ is equal to

(a) 38 (b) 36 (c) 34 (d) 30

63. If $x-\frac{1}{x}=4$, then $\left(x+\frac{1}{x}\right)$ is equal to

(a) $5\sqrt{2}$ (b) $2\sqrt{5}$
(c) $4\sqrt{2}$ (d) $4\sqrt{5}$

64. If $4b^2+\frac{1}{b^2}=2$, then the value of $8b^3+\frac{1}{b^3}$ is

(a) 0 (b) 1 (c) 2 (d) 5

YEAR : 2009

65. If $2^{2x-y} = 16$ and $2^{x+y} = 32$, the value of xy is

(a) 2 (b) 4 (c) 6 (d) 8

YEAR : 2010

66. If $\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$, then x is equal to

(a) –2 (b) 2 (c) – 1 (d) 1

67. If a and b be positive integers such that $a^2 - b^2 = 19$, then the value of a is

(a) 19 (b) 20 (c) 9 (d) 10

68. $\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}} = 2$ then x is equal to

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{5}{7}$ (d) $\frac{7}{5}$

69. If $x=\frac{\sqrt{3}}{2}$, then the value of $\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)$ is

(a) $-\sqrt{3}$ (b) – 1 (c) 1 (d) $\sqrt{3}$

70. If $4^{4x+1} = \frac{1}{64}$, then the value of x is

(a) $\frac{1}{2}$ (b) – 1 (c) $-\frac{1}{2}$ (d) $-\frac{1}{6}$

71. If $\frac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}-\sqrt{x-4}} = 2$ then x is equal to

(a) 2.4 (b) 3.2 (c) 4 (d) 5

72. If $\sqrt{2^x} = 256$, then the value of x is

(a) 14 (b) 16 (c) 18 (d) 20

73. If $(\sqrt{5})^7 \div (\sqrt{5})^5 = 5^p$, then the value of p is

(a) 5 (b) 2 (c) $\frac{3}{2}$ (d) 1

74. If 1.5 a = 0.04 b then $\frac{b-a}{b+a}$ is equal to

(a) $\frac{73}{77}$ (b) $\frac{77}{33}$ (c) $\frac{2}{75}$ (d) $\frac{75}{2}$

75. If $x=(\sqrt{2}+1)^{\frac{1}{3}}$, the value of $\left(x^3-\frac{1}{x^3}\right)$ is

(a) 0 (b) $-\sqrt{2}$ (c) + 2 (d) $3\sqrt{2}$

76. If $\frac{x^2-x+1}{x^2+x+1}=\frac{3}{2}$, then the value of $\left(x+\frac{1}{x}\right)$ is

(a) 4 (b) – 5 (c) 6 (d) 8

77. If $x = 3+\sqrt{8}$, then $x^2+\frac{1}{x^2}$ is equal to

(a) 38 (b) 36 (c) 34 (d) 30

78. If $x = 5+2\sqrt{6}$, then the value of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ is.

(a) $2\sqrt{2}$ (b) $3\sqrt{2}$
(c) $2\sqrt{3}$ (d) $3\sqrt{3}$

79. For $a > b$. If $a + b = 5$ and $ab = 6$, then the value of $(a^2 - b^2)$ is

(a) 1 (b) 3 (c) 5 (d) 7

80. If $x+\frac{9}{x}=6$, then the value of $\left(x^2+\frac{9}{x^2}\right)$ is

(a) 8 (b) 9 (c) 10 (d) 12

81. If $2p+\frac{1}{p}=4$, then value of $p^3+\frac{1}{8p^3}$ is

(a) 4 (b) 5 (c) 8 (d) 15

82. If $a^4 + b^4 = a^2b^2$, then $(a^6 + b^6)$ equals
(a) 0 (b) 1
(c) $a^2 + b^2$ (d) $a^2b^4 + a^4b^2$

83. If $x + \frac{1}{x} = 3$, then the value of $\frac{x^3 + \frac{1}{x}}{x^2 - x + 1}$ is:
(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{11}{2}$

84. If $a + \frac{1}{a} + 1 = 0\ (a \neq 0)$ then the value of $(a^4 - a)$ is:
(a) 0 (b) 1 (c) 2 (d) – 1

85. If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$, then the value of $x^4 + y^4 - 2x^2y^2$ is
(a) 24 (b) 18 (c) 16 (d) 12

86. If a = 11 and b = 9, then the value of $\left(\frac{a^2 + b^2 + ab}{a^3 - b^3}\right)$ is
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20

87. If p = 101, then the value of $\sqrt[3]{p\left(p^2 - 3p + 3\right) - 1}$ is
(a) 100 (b) 101
(c) 102 (d) 1000

88. If x = 19 and y = 18, then the value of $\frac{x^2 + y^2 + xy}{x^3 - y^3}$ is
(a) 1 (b) 37
(c) 324 (d) 361

89. If 50% of $(p - q)$ = 30% of $(p + q)$, then $p : q$ is equal to
(a) 5 : 3 (b) 4 : 1
(c) 3 : 5 (d) 1 : 4

90. If $x : y = 2 : 1$, then $(5x^2 - 13xy + 6y^2)$ is equal to
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 0 (b) $\frac{55}{4}$

YEAR : 2011

91. If $\frac{a}{3} = \frac{b}{2}$, then value of $\frac{2a + 3b}{3a - 2b}$ is
(a) $\frac{12}{5}$ (b) $\frac{5}{12}$ (c) 1 (d) $\frac{12}{7}$

92. If $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$?
(a) 1 (b) 2 (c) 3 (d) 4

93. If $\frac{2x - y}{x + 2y} = \frac{1}{2}$, then value of $\frac{3x - y}{3x + y}$ is:
(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (b) 1

94. If $x + \frac{1}{x} = 5$, then $\frac{2x}{3x^2 - 5x + 3}$ is equal to
(a) 5 (b) $\frac{1}{5}$ (c) 3 (d) $\frac{1}{3}$

95. If $\sqrt{1 - \frac{x^3}{100}} = \frac{3}{5}$, then x equals
(a) 2 (b) 4
(c) 16 (d) $(136)^{1/3}$

96. If $\sqrt{1 + \frac{x}{9}} = \frac{13}{3}$, then the value of x is
(a) $\frac{1439}{9}$ (b) 160
(c) $\frac{1443}{9}$ (d) 169

97. If $\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} = a + b\sqrt{6}$, then the values of a and b are respectively
(a) $\frac{9}{15}, -\frac{4}{15}$ (b) $\frac{3}{11}, \frac{4}{33}$
(c) $\frac{9}{10}, \frac{2}{5}$ (d) $\frac{3}{5}, \frac{4}{15}$

98. If $a * b = ab$, then the value of 5 *3 is
(a) 125 (b) 243 (c) 53 (d) 15

99. If $\sqrt{0.03 \times 0.3a} = 0.3 \times 0.3 \times \sqrt{b}$, value of $\frac{a}{b}$ is
(a) 0.009 (b) 0.03
(c) 0.9 (d) 0.08

100. If $x * y = (x + 3)^2 (y - 1)$, then the value of 5 * 4 is
(a) 192 (b) 182 (c) 180 (d) 172

101. If $9\sqrt{x} = \sqrt{12} + \sqrt{147}$, then x = ?
(a) 2 (b) 3 (c) 4 (d) 5

102. If $\sqrt{1 + \frac{x}{961}} = \frac{32}{31}$, then the value of x is
(a) 63 (b) 61 (c) 65 (d) 64

103. If $\sqrt{0.04 \times 0.4 \times a} = 0.004 \times 0.4 \times \sqrt{b}$, then the value of $\frac{a}{b}$ is
(a) 16×10^{-3} (b) 16×10^{-4}
(c) 16×10^{-5} (d) 16×10^{-6}

104. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = 3$ then $\frac{2a^2 + 3c^2 + 4e^2}{2b^2 + 3d^2 + 4f^2} = ?$
(a) 2 (b) 3 (c) 4 (d) 9

105. If $2x + \frac{1}{3x} = 5$. Find the value of
(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

106. If x varies inversely as $(y^2 - 1)$ and x is equal to 24 when y = 10, then the value of x when y = 5 is
(a) 99 (b) 12 (c) 24 (d) 100

107. If $x^2 + y^2 + 2x + 1 = 0$, then the value of $x^{31} + y^{35}$ is
(a) – 1 (b) 0 (c) 1 (d) 2

108. If $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$, then value of $\left(x + \frac{1}{x}\right)$ is:
(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) – 2

109. If a, b, c are real and $a^2 + b^2 + c^2 = 2(a - b - c) - 3$ then the value of $2a - 3b + 4c$ is
(a) – 1 (b) 0 (c) 1 (d) 2

110. If $(3a + 1)^2 + (b - 1)^2 + (2c - 3)^2 = 0$, then the value of $(3a + b + 2c)$ is equal to;
(a) 3 (b) – 1 (c) 2 (d) 5

111. The value of the expression
$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} +$
(a) 0 (b) 3 (c) $\frac{1}{3}$ (d) 2

112. If $(a-3)^2+(b-4)^2+(c-9)^2=0$, then the value of $\sqrt{a+b+c}$ is:

(a) –4 (b) 4 (c) ± 4 (d) ± 2

113. If $1.5x = 0.04y$, then the value of $\frac{y^2-x^2}{y^2+2xy+x^2}$ is

(a) $\frac{730}{77}$ (b) $\frac{73}{77}$ (c) $\frac{73}{770}$ (d) $\frac{74}{77}$

114. If $a^{\frac{1}{3}}=11$, then the value of a^2-331a is

(a) 1331331 (b) 1331000
(c) 1334331 (d) 1330030

115. If $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=4$, then the value of x^2+y^2 is

(a) 2 (b) 4 (c) 8 (d) 16

116. If $a^2+b^2=2$ and $c^2+d^2=1$ then the value of $(ad-bc)^2+(ac+bd)^2$ is

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

117. If $x=\frac{4ab}{a+b}$ $a \neq b$, the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ is

(a) a (b) b (c) 2 ab (d) 2

118. If $m+\frac{1}{m-2}=4$, find the value of $(m-2)^2+\frac{1}{(m-2)^2}$

(a) – 2 (b) 0 (c) 2 (d) 4

119. If $a^2+b^2+2b+4a+5=0$, then the value of $\frac{a-b}{a+b}$ is

(a) 3 (b) – 3 (c) $\frac{1}{3}$ (b) $-\frac{1}{3}$

120. If $x-y=\frac{x+y}{7}=\frac{xy}{4}$, the numerical value of xy is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

121. If $x+y+z=0$, then $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=?$

(a) $(xyz)2$ (b) $x^2+y^2+z^2$
(c) 9 (d) 3

122. If a+b+c = 0, then the value of $\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+a)}+\frac{1}{(c+a)(c+b)}$

(a) 1 (b) 0 (c) – 1 (d) – 2

123. If $a+b+c=0$, then the value of $\frac{a^2+b^2+c^2}{a^2-bc}$ is

(a) 0 (b) 1 (c) 2 (d) 3

124. If $x^2+y^2-4x-4y+8=0$, then the value of $x-y$ is

(a) 4 (b) – 4 (c) 0 (d) 8

125. If $x=b+c-2a$, $y=c+a-2b$, $z=a+b-2c$, then the value of $x^2+y^2-z^2+2xy$ is

(a) 0 (b) a +b +c
(c) a – b + c (d) a + b – c

126. If $x+\frac{1}{x}=\sqrt{3}$ then the value of $x^{18}+x^{12}+x^6+1$ is

(a) 0 (b) 1 (c) 2 (d) 3

127. If for two real constants a and b the expression ax^3+3x^2-8x+b is exactly divisible by $(x+2)$ and $(x-2)$, then

(a) $a=2, b=12$ (b) $a=12, b=2$
(c) $a=2, b=-12$ (d) $a=-2, b=12$

128. If $x^2-3x+1=0$, then the value of $x^3+\frac{1}{x^3}$ is

(a) 9 (b) 18 (c) 27 (d) 1

129. If $x+\frac{1}{4x}=\frac{3}{2}$, find the value of $8x^3+\frac{1}{8x^3}$.

(a) 18 (b) 36 (c) 24 (d) 16

130. If $x^4+\frac{1}{x^4}=119$ and $x>1$, then the value of $x^3+\frac{1}{x^3}$ is

(a) $6\sqrt{13}$ (b) $8\sqrt{13}$
(c) $13\sqrt{13}$ (d) $10\sqrt{13}$

131. If $3x+\frac{1}{2x}=5$, then the value of $8x^3+\frac{1}{27x^3}$ is:

(a) $118\frac{1}{2}$ (b) $30\frac{10}{27}$
(c) 0 (d) 1

132. If $x+y=z$, then the expression $x^3+y^3-z^3+3xyz$ will be equal to:

(a) 0 (b) $3xyz$
(c) $-3xyz$ (d) z3

133. If the sum of $\frac{a}{b}$ and its reciprocal is 1 and $a \neq 0$, $b \neq 0$, then the value of a^3+b^3 is

(a) 2 (b) –1 (c) 0 (d) 1

134. If $x=2-2^{1/3}+2^{2/3}$ then the value of $x^3-6x^2+18x+18$ is

(a) 22 (b) 33 (c) 40 (d) 45

135. If $a^3-b^3-c^3-3abc=0$, then

(a) a = b = c (b) $a+b+c=0$
(c) $a+c=b$ (d) $a=b+c$

136. If a = 2.361, b = 3.263 and c = 5.624, then the value of $a^3+b^3-c^3+3abc$ is

(a) $(p-q)(q-r)^3+(r-p)^3$
(b) 3 (p – q) (q–r) (r–p)
(c) 0
(d) 1

137. If p = 124,

$\sqrt[3]{p\left(p^2+3p+3\right)+1}=?$

$\sqrt[3]{p\left(p^2+3p+3\right)+1}=?$

(a) 5 (b) 7 (c) 123 (d) 125

138. If $x+\frac{1}{x}=2$ and x is real, then the value of $x^{17}+\frac{1}{x^{19}}$ is

(a) 1 (b) 0 (c) 2 (d) –2

139. If $x:y=3:4$, then the value of $\frac{5x-2y}{7x+2y}=?$

(a) $\frac{7}{25}$ (b) $\frac{7}{23}$ (c) $\frac{7}{29}$ (d) $\frac{7}{17}$

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140. If $\frac{2p}{p^2 - 2p+1} = \frac{1}{4}, p \neq 0$ then the value of $p + \frac{1}{p}$ is

(a) 4 (b) 5 (c) 10 (d) 12

141. If x +y = 2z then the value of $\frac{x}{x-z} + \frac{z}{y-z}$ is

(a) 1 (b) 3 (c) $\frac{1}{2}$ (d) 2

142. If $a^3b = abc = 180$, a, b, c are positive integers, then the value of c is

(a) 110 (b) 1 (c) 4 (d) 25

143. If a, b are rational number and $(a-1)\sqrt{2} + 3 = b\sqrt{2} + a$, the value of $(a+b)$ is

(a) – 5 (b) 3
(c) – 3 (d) 5

144. If $64^{x+1} = \frac{64}{4^x}$, then the value of x is

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) 2

145. If $ax^2 + bx + c = a(x-p)^2$, then the relation among a, b, c would be

(a) $abc = 1$ (b) $b^2 = ac$
(c) $b^2 = 4ac$ (d) $2b = a + c$

146. If $a + b + c + d = 1$, then the maximum value of $(1 + a)(1 + b)(1 + c)(1 + d)$ is

(a) 1 (b) $\left(\frac{1}{2}\right)^3$ (c) $\left(\frac{3}{4}\right)^3$ (d) $\left(\frac{5}{4}\right)^4$

147. If $a^2 + b^2 + c^2 + 3 = 2(a + b + c)$ then the value of $(a + b + c)$ is

(a) 2 (b) 3 (c) 4 (d) 5

148. If $x - \frac{1}{x} = 5$, then $x^2 + \frac{1}{x^2}$ is :

(a) 5 (b) 25 (c) 27 (d) 23

149. If $n = 7 + 4\sqrt{3}$, then the value of $\left(\sqrt{n} + \frac{1}{\sqrt{n}}\right)$ is:

(a) $2\sqrt{3}$ (b) 4
(c) – 4 (d) $-2\sqrt{3}$

150. If $x = \sqrt{3} + \sqrt{2}$, then the value of $\left(x + \frac{1}{x}\right)$ is

(a) $2\sqrt{2}$ (b) $2\sqrt{3}$
(c) 2 (d) 3

151. If p + q =10 and pq = 5, then the numerical value of $\frac{p}{q} + \frac{q}{p}$ will be

(a) 16 (b) 20 (c) 22 (d) 18

152. If $x = 3 + 2\sqrt{2}$ and xy = 1, then the value of $\frac{x^2 + 3xy + y^2}{x^2 - 3xy + y^2}$ is

(a) $\frac{30}{31}$ (b) $\frac{70}{31}$ (c) $\frac{35}{31}$ (d) $\frac{37}{31}$

153. If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$, then

(a) $\frac{x-y}{b-a} = \frac{y-z}{c-b} = \frac{z-x}{a-c}$

(b) $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

(c) $\frac{x-y}{c} = \frac{y-z}{b} = \frac{z-x}{c}$

(d) None of the above is true

154. If $x - y = 2$, $xy = 24$, then the value of $(x^2 + y^2)$ is

(a) 25 (b) 36 (c) 63 (d) 52

155. If the expression $\frac{x^2}{y^2} + tx + \frac{y^2}{4}$ is a perfect square, then the values of t is

(a) ± 1 (b) ± 2 (c) 0 (d) ± 3

156. If $a = x + y$, $b = x - y$, $c = x + 2y$, then $a^2 + b^2 + c^2 - ab - bc - ca$ is

(a) $4y^2$ (b) $5y^2$
(c) $6y^2$ (d) $7y^2$

157. If $x + \frac{1}{x} = 2, x \neq 0$ then value of $x^2 + \frac{1}{x^3}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

158. If $\left(x + \frac{1}{x}\right)^2 = 3$ then the value of $(x^{72} + x^{66} + x^{54} + x^{24} + x^6 + 1)$

(a) 0 (b) 1 (c) 84 (d) 206

159. If $a + \frac{1}{a} = \sqrt{3}$, then the value of $a^6 - \frac{1}{a^6} + 2$ will be

(a) 1 (b) 2 (c) $3\sqrt{3}$ (d) 5

160. If $x^3 + y^3 = 35$ and $x + y = 5$, then the value of $\frac{1}{x} + \frac{1}{y}$ will be:

(a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) 6 (d) $\frac{2}{3}$

161. If $a^3 - b^3 = 56$ and $a - b = 2$ then value of $a^2 + b^2$ will be:

(a) 48 (b) 20 (c) 22 (d) 5

162. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ is, then $\frac{a}{b} + \frac{b}{a} = ?$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

163. If $x + \frac{1}{x} = 5$, then the value of $\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$

(a) $\frac{43}{23}$ (b) $\frac{47}{21}$ (c) $\frac{41}{23}$ (d) $\frac{45}{21}$

164. If x is real, $x + \frac{1}{x} \neq 0$ and $x^3 + \frac{1}{x^3} = 0$, then the value of $\left(x + \frac{1}{x}\right)^4$ is

(a) 4 (b) 9 (c) 16 (d) 25

165. If $x + \frac{1}{x} = 3$, then the value of $\left(x^5 + \frac{1}{x^5}\right)$ is

(a) 322 (b) 126
(c) 123 (d) 113

166. If $x - \frac{1}{x} = 3$, then value of $x^3 - \frac{1}{x^3}$ is

(a) 32 (b) 36 (c) 40 (d) 49

167. If $m^4 + \frac{1}{m^4} = 119$, then $m - \frac{1}{m} = ?$

(a) ± 3 (b) 4 (c) ± 2 (d) ± 1

168. If $x + y + z = 6$, then the value of $(x-1)^3 + (y-2)^3 + (z-3)^3$ is

(a) $3(x-1)(y+2)(z-3)$
(b) $3(x+1)(y-2)(z-3)$
(c) $3(x-1)(y-2)(z+3)$
(d) $3(x-1)(y-2)(z-3)$

169. If $a+b+c=6$, $a^2+b^2+c^2=14$ and $a^3+b^3+c^3=36$, then the value of abc is
(a) 3 (b) 6 (c) 9 (d) 12

170. If $a+b=1$ and $a^3+b^3+3ab=k$, then the value of k is
(a) 1 (b) 3 (c) 5 (d) 7

171. If $a=34$, $b=c=33$, then the value of $a^3+b^3+c^3-3abc$ is
(a) 0 (b) 111 (c) 50 (d) 100

172. If $(2^x)(2^y)=8$ and $(9^x)(3^y)=81$, then (x, y) is:
(a) (1,2) (b) (2,1)
(c) (1,1) (d) (2,2)

173. The lines $2x+y=5$ and $x+2y=4$ intersect at the point:
(a) (1,2) (b) (2,1)
(c) $\left(\frac{5}{2},0\right)$ (d) (0,2)

174. If y : x = 4 : 15, then the value of $\left(\frac{x-y}{x+y}\right)$ is
(a) $\frac{11}{19}$ (b) $\frac{19}{11}$ (c) $\frac{4}{11}$ (d) $\frac{15}{19}$

175. The expression x^4-2x^2+k will be a perfect square when the value of k is
(a) 2 (b) 1 (c) – 1 (d) – 2

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176. If $(x-3)^2+(y-5)^2+(z-4)^2=0$ then the value of
$\frac{x^2}{9}+\frac{y^2}{25}+\frac{z^2}{16}$ is
(a) 12 (b) 9 (c) 3 (d) 1

177. x varies inversely as square of y. Given that $y=2$ for $x=1$, the value of x for $y=6$ will be equal to
(a) 3 (b) 9 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

178. If $a^2+b^2+c^2+3=2(a-b-c)$ then the value of $2a-b+c$ is :
(a) 3 (b) 4 (c) 0 (d) 2

179. If $x^2-y^2=80$ and $x-y=8$, then the average of x and y is
(a) 2 (b) 3 (c) 4 (d) 5

180. The third proportional to $\left(\frac{x}{y}+\frac{y}{x}\right)$ and $\sqrt{x^2+y^2}$ is
(a) xy (b) $\sqrt{xy}$
(c) $\sqrt[3]{xy}$ (d) $\sqrt[4]{xy}$

181. If $\frac{4x}{3}+2P=12$ for what value of P, x = 6 ?
(a) 6 (b) 4 (c) 2 (d) 1

182. The value of $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}$ is
(a) $5\sqrt{3}-8$ (b) $5\sqrt{3}+8$
(c) $8\sqrt{3}+5$ (d) $8\sqrt{3}-5$

183. Let $a=\sqrt{6}-\sqrt{5}$,
$b=\sqrt{5}-2, c=2-\sqrt{3}$ Then point out the correct alternative among the four alternatives given below.
(a) $b<a<c$ (b) $a<c<b$
(c) $b<c<a$ (d) $a<b<c$

184. If $x=\frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, the value of $\frac{x+\sqrt{20}}{x-\sqrt{20}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$ is
(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{5}$

185. If $x=5-\sqrt{21}$, then the value of $\frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$ is
(a) $\frac{1}{\sqrt{2}}\left(\sqrt{3}-\sqrt{7}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$
(b) $\frac{1}{\sqrt{2}}\left(\sqrt{7}+\sqrt{3}\right)$ (d) $\frac{1}{\sqrt{2}}\left(7+\sqrt{3}\right)$

186. If $6x-5y=13, 7x+2y=23$ then $11x+18y=$?
(a) – 15 (b) 51 (c) 33 (d) 15

187. The value of $\left(x^{b+c}\right)^{b-c}\left(x^{c+a}\right)^{c-a}\left(x^{a+b}\right)^{a-b}$
(a) 1 (b) 2 (c) – 1 (d) 0

188. If $\frac{x}{a}=\frac{1}{a}-\frac{1}{x}$, then the value of $x-x^2$ is:
(a) $-a$ (b) $\frac{1}{a}$ (c) a (d) $-\frac{1}{a}$

189. If $x+\frac{1}{x}=99$, find the value of
(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

190. If $\frac{4x-3}{x}+\frac{4y-3}{y}+\frac{4z-3}{z}=0$ then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is
(a) 9 (b) 3 (c) 4 (d) 6

191. If $\frac{xy}{x+y}=a, \frac{xz}{x+z}=b$ and $\frac{yz}{y+z}=c$, where a, b, c are all non -zero numbers, then x equals to
(a) $\frac{2abc}{ab+bc-ac}$ (b) $\frac{2abc}{ab+ac-bc}$
(c) $\frac{2abc}{ac+bc-ab}$ (d) $\frac{2abc}{ab+bc-ac}$

192. If x and y are positive real numbers and xy =8, then the minimum value of $2x+y$ is
(a) 9 (b) 17 (c) 10 (d) 8

193. If the expression x^2+x+1 is written in the form $\left(x+\frac{1}{2}\right)^2+q^2$, then the possible values of q are
(a) $\pm\frac{1}{3}$ (b) $\pm\frac{\sqrt{3}}{2}$
(c) $\pm\frac{2}{\sqrt{3}}$ (d) $\pm\frac{1}{2}$

194. If $a^2-4a-1=0$, then value of $a^2+\frac{1}{a^2}+3a-\frac{3}{a}$ is
(a) 25 (b) 30 (c) 35 (d) 40

195. The minimum value of $(x-2)(x-9)$ is
(a) $-\frac{11}{4}$ (b) $\frac{49}{4}$ (c) 0 (d) $-\frac{49}{4}$

196. One of the factors of the expression $4\sqrt{3}x^2+5x-2\sqrt{3}$ is :
(a) $4x+\sqrt{3}$ (b) $4x+3$
(c) $4x-3$ (d) $4x-\sqrt{3}$

197. If $\sqrt{x}=\sqrt{3}-\sqrt{5}$, then the value of $x^2-16x+6$ is
(a) 0 (b) – 2 (c) 2 (d) 4

198. If $x^2=y+z, y^2=z+x, z^2=x+y$, then the value of $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$
(a) – 1 (b) 1 (c) 2 (d) 41

199. If a +b +c = 0, then the value of $\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)$

(a) 8 (b) – 3 (c) 9 (d) 0

200. If a, b, c are non - zero $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$, then the value of abc is:

(a) – 1 (b) 3 (c) – 3 (d) 1

201. If $a + b + c = 2s$, then $\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$

(a) $a^2+b^2+c^2$ (b) 0
(c) 1 (d) 2

202. If $x = 3+2\sqrt{2}$, the value of $x^2+\frac{1}{x^2}$ is

(a) 36 (b) 30 (c) 32 (d) 34

203. If $x\left(3-\frac{2}{x}\right)=\frac{3}{x}$, then the value of $x^2+\frac{1}{x^2}$ is

(a) $2\frac{1}{9}$ (b) $2\frac{4}{9}$ (c) $3\frac{1}{9}$ (d) $3\frac{4}{9}$

204. If $x^2 - 3x + 1 = 0$, then the value of $x^2+x+\frac{1}{x}+\frac{1}{x^2}$ is

(a) 10 (b) 2 (c) 6 (d) 8

205. If $a^2+b^2 = 5ab$, the value of $\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)$ is:

(a) 32 (b) 16 (c) 23 (d) – 23

206. If $xy + yz + zx = 0$, then $\left(\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}\right)(x, y, z \neq 0)$

(a) 3 (b) 1
(c) $x + y + z$ (d) 0

207. If a +b +c = 9 (where a, b, c are real numbers), then the minimum value of $a^2+b^2+c^2$ is

(a) 100 (b) 9 (c) 27 (d) 81

208. If $a^2+b^2+4c^2 = 2(a+b-2c) - 3$ and a, b, c are real, then the value of $(a^2+b^2+c^2)$ is

(a) 3 (b) $3\frac{1}{4}$ (c) 2 (d) $2\frac{1}{4}$

209. If $\frac{x-a^2}{b+c}+\frac{x-b^2}{c+a}+\frac{x-c^2}{a+b}$ $= 4(a+b+c)$, then x

(a) $(a+b+c)^2$ (b) $a^2+b^2+c^2$
(c) $ab+bc+ca$
(d) $a^2+b^2+c^2-ab-bc-ca$

210. Number of solutions of the two equations $4x - y = 2$ and $2x - 8y + 4 = 0$ is

(a) zero (b) one
(c) two (d) Infinitely many

211. If $\frac{a}{b}=\frac{4}{5}$ and $\frac{b}{c}=\frac{15}{16}$, then $\frac{18c^2-7a^2}{45c^2+20a^2}$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$
(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

212. If $x \neq 0, y \neq 0$ and $z \neq 0$ and $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}$ then the relation among x, y, z is

(a) $x + y + z = 0$
(b) $x + y = z$
(c) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$
(d) $x = y = z$

213. If $a+b = 1$, $c + d = 1$ and $a-b=\frac{d}{c}$, then the value of c^2-d^2

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) 1 (d) – 1

214. If $x=3t$, $y = \frac{1}{2}(t+1)$, then the value of t for which $x = 2y$ is

(a) 1 (b) $\frac{1}{2}$ (c) – 1 (d) $\frac{2}{3}$

215. If $x^2+\frac{1}{5}x+a^2$ is a perfect square, then a is

(a) $\frac{1}{100}$ (b) $\pm\frac{1}{10}$ (c) $\frac{1}{10}$ (d) $-\frac{1}{10}$

216. Find the value of x for which the expression $2-3x-4x^2$ has the greatest value.

(a) $-\frac{41}{16}$ (b) $\frac{3}{8}$ (c) $-\frac{3}{8}$ (d) $\frac{41}{16}$

217. If $(x-1)$ and $(x+3)$ are the factors of $x^2+k_1x+k_2$ then

(a) $k_1 = -2$, $k_2 = -3$
(b) $k_1 = 2$, $k_2 = -3$
(c) $k_1 = -2$, $k_2 = 3$
(d) $k_1 = -2$, $k_2 = 3$

218. If $\frac{5x}{2x^2+5x+1}=\frac{1}{3}$, then the value of $\left(x+\frac{1}{2x}\right)$

(a) 15 (b) 10 (c) 20 (d) 5

219. If $x>1$ and $x^2+\frac{1}{x^2}=83$ then $x^3-\frac{1}{x^3}$ is

(a) 764 (b) 750 (c) 756 (d) 760

220. If $\left(a+\frac{1}{a}\right)^2 = 3$, then $a^3+\frac{1}{a^3} = ?$

(a) $2\sqrt{3}$ (b) 2 (c) $3\sqrt{3}$ (d) 0

221. If $\frac{x}{x^2-2x+1}=\frac{1}{3}$, then the value of $x^3+\frac{1}{x^3}$ is:

(a) 64 (b) 110 (c) 81 (d) 124

222. If $\left(x+\frac{1}{x}\right)=4$, then the value of $x^4+\frac{1}{x^4}$ is:

(a) 64 (b) 194 (c) 81 (d) 124

223. If $x + y + z = 6$ and $x^2+y^2+z^2 = 20$ then the value of $x^3+y^3+z^3-3xyz$ is

(a) 64 (b) 70 (c) 72 (d) 76

224. If $x = 1-\sqrt{2}$, the value of $\left(x-\frac{1}{x}\right)^3$ is:

(a) – 8 (b) 8
(c) $2\sqrt{2}$ (b) 1

225. If $x = a - b$, $y = b - c$, $z = c - a$, then the numerical value of the algebraic expression $x^3+y^3+z^3-3xyz$ will be

(a) $a + b + c$ (b) 0
(c) $4(a+b+c)$ (d) $3abc$

226. If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of $x^3 + y^3$ is :

(a) 950 (b) 730
(c) 650 (d) 970

227. If $(x-a)(x-b) = 1$ and $a - b + 5 = 0$, then the value of $(x-a)^3 - \frac{1}{(x-a)^3}$ is

(a) –125 (b) 1
(c) 125 (d) 140

228. If $a^2 + b^2 + c^2 = 2(a-b-c) - 3$ then the value of $4a - 3b + 5c$ is

(a) 2 (b) 3 (c) 5 (d) 6

229. If $2x + \frac{2}{x} = 3$, then the value of $x^3 + \frac{1}{x^3} + 2$ is

(a) $-\frac{9}{8}$ (b) $-\frac{25}{8}$ (c) $\frac{7}{8}$ (d) 11

230. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 83$ then the value of $a^3 + b^3 + c^3 - 3abc$

(a) 200 (b) 180
(c) 190 (d) 210

231. If $a - b = 3$ and $a^3 - b^3 = 27$ then $|a+b|$ is equal to

(a) 3 (b) 5 (c) 7 (d) 9

232. If $x + \frac{1}{x+1} = 1$, then $(x+1)^5 + \frac{1}{(x+1)^5}$ equals

(a) 1 (b) 2 (c) 4 (d) 8

233. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ is equal to

(a) $a + b + c$ (d) abc
(c) $2abc$ (d) $3abc$

234. If $x = y = 333$ and $z = 334$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is

(a) 0 (b) 667
(c) 1000 (d) 2334

235. Out of the given responses one of the factors of $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$ is

(a) (a + b)(a–b) (b) (a + b)(a+b)
(c) (a – b)(a–b) (d) (b – c)(b–c)

236. If $a = \frac{b^2}{b-a}$ then the value of $a^3 + b^3$ is

(a) $6ab$ (b) 0
(c) 1 (d) 2

237. If $p - 2q = 4$, then the value of $p^3 - 8q^3 - 24pq - 64$ is

(a) 2 (b) 0 (c) 3 (d) – 1

238. If $x = -1$, then the value of

$$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x} - 1$$

(a) 1 (b) 0 (c) – 2 (d) – 1

239. If $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$ and a, b, c are rational numbers then a + b + c is equal to

(a) 0 (b) 1 (c) 2 (d) 3

240. If $x = \sqrt[3]{2 + \sqrt{3}}$, then the value of $x^3 + \frac{1}{x^3}$ is

(a) 8 (b) 9 (c) 2 (d) 4

241. If $x = \sqrt[3]{5} + 2$, then the value of $x^3 - 6x^2 + 12x - 13$

(a) – 1 (b) 1 (c) 2 (d) 0

242. The simplest form of the expression

$$\frac{p^2 - p}{2p^3 + p^2} + \frac{p^2 - 1}{p^2 + 3p} + \frac{p^2}{p+1}$$

(a) $2p^2$ (b) $\frac{1}{2p^2}$
(c) p +3 (d) $\frac{1}{p+3}$

243. If $x + \frac{1}{x} = 2$, then the value of $\left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right)$ is

(a) 20 (b) 4 (c) 8 (d) 16

244. If a,b,c be all positive integers then the least positive value of $a^3 + b^3 + c^3 - 3abc$ is.

(a) 0 (b) 2 (c) 4 (d) 3

245. When $f(x) = 12x^3 - 13x^2 - 5x + 7$ is divided by $(3x + 2)$, then the remainder is

(a) 2 (b) 0 (c) – 1 (d) 1

246. If the equation $2x^2 - 7x + 12 = 0$ has two roots α and β, then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is

(a) $\frac{7}{2}$ (b) $\frac{1}{24}$ (c) $\frac{7}{24}$ (d) $\frac{97}{24}$

247. If $x^3 + \frac{3}{x} = 4(a^3 + b^3)$ and $3x + \frac{1}{x^3} = 4(a^3 - b^3)$, then $a^2 - b^2$ is equal to

(a) 4 (b) 0 (c) 1 (d) 2

248. If $x = 6 + \frac{1}{x}$, then the value of $x^4 + \frac{1}{x^4} + 2$ is

(a) 1448 (b) 1442
(c) 1444 (d) 1446

249. The graph of $2x + 1 = 0$ and $3y - 9 = 0$ intersect at the point

(a) $\left(-\frac{1}{2}, -3\right)$ (b) $\left(-\frac{1}{2}, 3\right)$
(c) $\left(\frac{1}{2}, -3\right)$ (d) None of these

250. If $x^2 + 9y^2 = 6xy$, then $x : y$ is

(a) 1:3 (b) 3:2
(c) 3:1 (d) 2:3

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251. The term to be added to $121a^2 + 64b^2$ to make a perfect square is

(a) $176\ ab$ (b) $276\ a^2b$
(c) $178\ ab$ (d) $188\ b^2a$

252. If $a = 2 + \sqrt{3}$, then the value of $\left(a^2 + \frac{1}{a^2}\right)$

(a) 12 (b) 14 (c) 16 (d) 10

253. For what value of k the expression $p + \frac{1}{4} + \sqrt{p} + k^2$ is perfect square?

(a) 0 (b) $\pm\frac{1}{4}$ (c) $\pm\frac{1}{8}$ (d) $\pm\frac{1}{2}$

254. If $\frac{b-c}{a} + \frac{a+c}{b} + \frac{a-b}{c} = 1$ and $a - b + c \neq 0$ then which one of the following relations is true?

(a) $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$ (b) $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$
(c) $\frac{1}{b} = \frac{1}{a} - \frac{1}{c}$ (d) $\frac{1}{b} = \frac{1}{a} + \frac{1}{c}$

255. The reciprocal of $x+\frac{1}{x}$ is

(a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x+1}$

(c) $x-\frac{1}{x}$ (d) $\frac{1}{x}+x$

256. If a, b, c are positive and a+b+c =1, then the least value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is

(a) 9 (b) 5 (c) 3 (d) 1

257. If $a\left(2+\sqrt{3}\right)=b\left(2-\sqrt{3}\right)=1$, then the value of $\frac{1}{a^2+1}+\frac{1}{b^2+1}$

(a) – 1 (b) 1 (c) 4 (d) 9

258. If $\left(2+\sqrt{3}\right)a=\left(2-\sqrt{3}\right)b=1$ then the value of $\frac{1}{a}+\frac{1}{b}$ is

(a) 1 (b) 2 (c) $2\sqrt{3}$ (d) 4

259. If $a+\frac{1}{b}=b+\frac{1}{c}=c+\frac{1}{a}$ (a ≠ b ≠ c), then the value of abc is

(a) ± 1 (b) ± 2 (c) 0 (d) $\pm\frac{1}{2}$

260. If $\frac{x}{y}=\frac{4}{5}$, then the value of $\left(\frac{4}{7}+\frac{2y-x}{2y+x}\right)$ is

(a) $\frac{3}{7}$ (b) $1\frac{1}{7}$ (c) 1 (d) 2

261. If $(x-2)$ is a factor of $x^2+3Qx-2Q$, then the value of Q is

(a) 2 (b) – 2 (c) 1 (d) – 1

262. If $a+b=12$, $ab=22$, then (a^2+b^2) is equal to

(a) 188 (b) 144 (c) 34 (d) 100

263. If $x=\sqrt{3}-\frac{1}{\sqrt{3}}$ and $y=\sqrt{3}+\frac{1}{\sqrt{3}}$, then the value of $\frac{x^2}{y}+\frac{y^2}{x}$ is

(a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) $16\sqrt{3}$ (d) $2\sqrt{3}$

264. If x^2+ax+b is a perfect square, then which one of the following relations between a and b is true

(a) $a^2=b$ (b) $a^2=4b$

(c) $b=4a$ (d) $b^2=a$

265. If a + b + c +d =4, then find the value of $\frac{1}{(1-a)(1-b)(1-c)}+\frac{1}{(1-b)(1-c)(1-d)}+\frac{1}{(1-c)(1-d)(1-a)}+\frac{1}{(1-d)(1-a)(1-b)}$

(a) 0 (b) 5 (c) 1 (d) 4

266. If $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}=0$, then a relation among a, b, c is

(a) a + b + c = 0

(b) $(a+b+c)^3=27abc$

(c) a +b +c = 3abc

(d) $a^3+b^3+c^3=0$

267. If $x-\frac{1}{x}=1$, then the value of $\frac{x^4-\frac{1}{x^2}}{3x^2+5x-3}$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 0

268. If $x+y=15$, then $(x-10)^3+(y-5)^3$ is

(a) 25 (b) 125

(c) 625 (d) 0

269. If $x^2+\frac{1}{x^2}=66$, then the value of $\frac{x^2-1+2x}{x}=?$

(a) ±8 (b) 10, – 6

(c) 6, – 10 (d) ±4

270. If $a^2+a+1=0$, then the value of a^9 is

(a) 2 (b) 3 (c) 1 (d) 0

271. If $x+\frac{2}{x}=1$, then the value of $\frac{x^2+x+2}{x^2(1-x)}$ is

(a) 1 (b) – 1 (d) 2 (d) – 2

272. If $x=-2k$ and $y=1-3k$, then for what value of k, will be $x=y$?

(a) 0 (b) 1 (c) – 1 (d) 2

273. Find the value of

$\sqrt{\left(x^2+y^2+z\right)\left(x+y-3z\right)}+\sqrt[3]{xy^3z^2}$

when $x=+1$, $y=-3$, $z=-1$,

(a) 1 (b) 0 (c) – 1 (d) $\frac{1}{2}$

274. If $x+\frac{1}{x}=5$, then $x^6+\frac{1}{x^6}$ is

(a) 12098 (b) 12048

(c) 14062 (d) 12092

275. If $x^2-3x+1=0$, then the value of $\frac{x^6+x^4+x^2+1}{x^3}$ will be

(a) 18 (b) 15 (c) 21 (d) 30

276. If $\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$ & $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}=0$, where p,q,r and a, b, c are non-zero, then the value of $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}$

(a) – 1 (b) 0 (c) 1 (d) 2

277. If x is a rational number and $\frac{(x+1)^3-(x-1)^3}{(x+1)^2-(x-1)^2}=2$, then the sum of numerator and denominator of x is:

(a) 3 (b) 4 (c) 5 (d) 7

278. If $x=\sqrt{5}+2$, then the value $\frac{2x^2-3x-2}{3x^2-4x-3}$ is equal to

(a) 0.1785 (b) 0.525

(c) 0.625 (d) 0.785

279. If a = 2.234, b= 3.121 and c = –5.355, then the value of $a^3+b^3+c^3-3abc$ is

(a)– 1 (b) 0 (c) 1 (d) 2

280. If $x^2+y^2+1=2x$, then the value of x^3+y^5 is

(a) 2 (b) 0 (c) – 1 (d) 1

281. If $3(a^2+b^2+c^2)=(a+b+c)^2$ then the relation between a, b and c is

(a) a = b = c (b) a = b ≠ c

(c) a < b < c (d) a > b > c

282. If $x(x-3)=-1$, then the value of $x^3(x^3-18)$ is

(a) –1 (b) 2 (c) 1 (d) 0

283. The factors of ($a^2 + 4b^2 + 4b - 4ab - 2a - 8$) are
(a) (a – 2b – 4) (a – 2b + 2)
(b) (a – b – 2) (a + 2b +2)
(c) (a + 2b – 4) (a + 2b + 2)
(d) (a + 2b– 4) (a – 2b + 2)

284. The value of
$$\frac{1}{a^2+ax+x^2}-\frac{1}{a^2-ax+x^2}+\frac{2ax}{a^4+a^2x^2+x^4}$$ is
(a) 2 (b) 1 (c) – 1 (d) 0

285. If $x = 11$, then the value of $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$ is
(a) 5 (b) 10 (c) 15 (d) 20

286. Which one is not an example of an equality relation of two expressions in x :
(a) $(x+3)^2 = x^2 + 6x + 9$
(b) $(x+2y)^3 = x^3 + 8y^3 + 6xy(x+2y)$
(c) $(x+2)^2 = x^2 + 2x + 4$
(d) $(x+3)(x-3) = x^2 - 9$

287. If $x = 997$, $y = 998$ and $z = 999$ then the value of $x^2 + y^2 + z^2 - xy - yz - zx$ is
(a) 0 (b) 1 (c) – 1 (d) 3

288. If $x+\frac{1}{x}=3$, then the value of
$$\frac{3x^2-4x+3}{x^2-x+1}$$ is
(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{3}$

289. If $x = 3+2\sqrt{2}$,
then $\frac{x^6+x^4+x^2+1}{x^3}$ is equal to
(a) 216 (b) 192 (c) 198 (d) 204

290. If $x = p + \frac{1}{p}$ and $y = p - \frac{1}{p}$ then the value of $x^4 - 2x^2y^2 + y^4$
(a) 24 (b) 4 (c) 16 (d) 8

291. If a +b +c = 0, then the value of $(a+b-c)^2+(b+c-a)^2+(c+a-b)^2$ is
(a) 0 (b) 8abc
(c) $4(a^2+b^2+c^2)$ (d) $4(ab+bc+ca)$

292. If $p^3 + 3p^2 + 3p = 7$, then the value of p^2+2p is
(a) 4 (b) 3 (c) 5 (d) 6

293. If $x = 2015$, $y = 2014$ and $z = 2013$, then value of $x^2+y^2+z^2 - xy - yz - zx$ is
(a) 3 (b) 4 (c) 6 (d) 2

294. If $3a^2 = b^2 \neq 0$, then the value of
$$\frac{(a+b)^3-(a-b)^3}{(a+b)^2+(a-b)^2}$$ is
(a) $\frac{3b}{2}$ (b) b (c) $\frac{b}{2}$ (d) $\frac{2b}{3}$

295. If $x > 1$ and $x+\frac{1}{x}=2\frac{1}{12}$, then the value of $x^4-\frac{1}{x^4}$ is
(a) $\frac{58975}{20736}$ (b) $\frac{59825}{20736}$
(c) $\frac{57985}{20736}$ (d) $\frac{57895}{20736}$

296. The value of $\frac{4x^3-x}{(2x+1)(6x-3)}$ when $x = 9999$ is
(a) 1111 (b) 2222
(c) 3333 (d) 6666

297. If $a^3+b^3=9$ and $a+b=3$, then the value of $\frac{1}{a}+\frac{1}{b}$ is
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) – 1

298. If $t^2 - 4t + 1 = 0$, then the value of $t^3+\frac{1}{t^3}$ is:
(a) 44 (b) 48 (c) 52 (d) 64

299. If $\sqrt[3]{a}+\sqrt[3]{b}=\sqrt[3]{c}$, then the simplest value of $(a+b-c)^3 + 27abc$ is
(a) – 1 (b) 3 (c) – 3 (d) 0

300. If 4x+5y =83 and 3x : 2y =21: 22, then $(y - x)$ equals
(a) 3 (b) 4 (c) 7 (d) 11

301. If $x=\sqrt[3]{a+\sqrt{a^2+b^3}}+\sqrt[3]{a-\sqrt{a^2+b^3}}$,
then $x^3 + 3bx$ is equal to
(a) 0 (b) a (c) $2a$ (d) 1

302. If $\frac{x^{24}+1}{x^{12}}=7$ then the value of
$$\frac{x^{72}+1}{x^{36}}$$
(a)343 (b) 433
(c) 432 (d) 322

(SSS CGL 16–08–2015,Morning)

303. If $x = 2$ then the value of $x^3 + 27x^2 + 243x + 631$
(a) 1321 (b) 1233
(c) 1231 (d) 1211

(SSS CGL 16–08–2015,Morning)

304. If $5x + 9y = 5$ and $125x^3 + 729y^3 = 120$,then the value of the product of x and y is
(a) 45 (b) 1/9 (c) 1/135 (d)135

(SSS CGL 16–08–2015,Morning)

305. If $x^2 + y^2 + z^2 = 2(x + z - 1)$, then the value of; $x^3 + y^3 + z^3 = ?$
(a) – 1 (b) 2 (c) 0 (d) 1

(SSS CGL 16–08–2015,Evening)

306. If $x+\frac{1}{x}=1$, then the value of
$$\frac{2}{x^2-x+2}=?$$
(a) 2/3 (b) 2 (c) 1 (d) 4

(SSS CGL 16–08–2015,Evening)

307. If $x=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $y=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$,
then the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2}=?$
(a) $\frac{65}{63}$ (b) $\frac{67}{65}$ (c) $\frac{69}{67}$ (d) $\frac{63}{61}$

(SSS CGL 16–08–2015, Evening)

308. If $4a-\frac{4}{a}+3=0$, then the value of
$$a^3-\frac{1}{a^3}+3=?$$
(a) $\frac{7}{16}$ (b) $\frac{3}{16}$ (c) $\frac{21}{64}$ (d) $\frac{21}{16}$

(SSS CGL 16–08–2015,Evening)

309. Simplified value of
$$\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)\left(1+\frac{1}{10+\frac{1}{10}}\right)-\left(1-\frac{1}{10+\frac{1}{10}}\right)\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]\div\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)+\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]$$
(a) $\frac{20}{101}$ (b) $\frac{100}{101}$ (c) 2 (d) $\frac{90}{101}$

(SSS CGL 16–08–2015,Evening)

310. If $x = z = 225$ and $y = 226$ then the value of: $x^3 + y^3 + z^3 - 3xyz$

(a) 765 (b) 676
(c) 674 (d) 576

(SSS CGL 16–08–2015,Evening)

311. If a + b – c = 14 then the value of: $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$

(a) 28 (b) 0 (c) 14 (d) 7

(SSS CGL 16–08–2015,Evening)

312. If $x^2+x=5$ then the value of:

$$(x+3)^3 + \frac{1}{(x+3)^3}$$

(a) 140 (b) 110 (c) 130 (d) 120

(SSS CGL 16–08–2015,Evening)

313. If m = – 4, n = –2, then the value of $m^3 - 3m^2 + 3m + 3n + 3n^2 + n^3$ is

(a) 124 (b) –124
(c) 126 (d) – 126

(SSS CGL 09–08–2015,Morning)

314. $2x - ky + 7 = 0$ and $6x - 12y + 15 = 0$ has no solution for:

(a) k = – 4 (b) k =4
(c) k = 1 (d) k = –1

(SSS CGL 09–08–2015,Morning)

315. If $x = 332$, $y = 333$, $z = 335$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is

(a) 7000 (b) 8000
(c) 9000 (d) 10000

(SSS CGL 09–08–2015,Morning)

316. If $2 + x\sqrt{3} = \frac{1}{2+\sqrt{3}}$, then the simplest value of x is:

(a) 1 (b) –2 (c) 2 (d) –1

(SSS CGL 09–08–2015, Morning)

317. $\frac{m-a^2}{b^2+c^2} + \frac{m-b^2}{c^2+a^2} + \frac{m-c^2}{a^2+b^2} = 3$, then the value of m is:

(a) $a^2 + b^2$ (b) $a^2 + b^2 + c^2$
(c) $a^2 - b^2 - c^2$ (d) $a^2 + b^2 - c^2$

(SSS CGL 09–08–2015,Morning)

318. If m – 5n = 2, then the value of $(m^3 - 125n^3 - 30mn)$ is:

(a) 6 (b) 7 (c) 8 (d) 9

319. Given that $x^3 + y^3 = 72$ and $xy = 8$ with $x > y$, then the value of $x - y$ is:

(a) 4 (b) 2 (c) – 2 (d) – 4

(SSS CGL 09–08–2015,Evening)

320. If $x = \sqrt{a\sqrt[3]{b\sqrt{a\sqrt[3]{b.....\infty}}}}$, then the value of x is:

(a) $\sqrt[5]{ab^3}$ (b) $\sqrt[3]{a^5b}$
(c) $\sqrt[3]{a^3b}$ (d) $\sqrt[5]{a^3b}$

(SSS CGL 09–08–2015, Evening)

321. If $x + \frac{1}{x} = 2$, then the value of $x^{12} - \frac{1}{x^{12}}$ is:

(a) – 4 (b) 4 (c) 2 (d 0

(SSS CGL 09–08–2015,Evening)

322. If $x + \frac{1}{x} = 1$, then the value of $\frac{x^2+3x+1}{x^2+7x+1}$ is:

(a) 1/2 (b) 3/7
(c) 2 (d) 3

(SSS CGL 09–08–2015,Evening)

323. If x +(1/x) =2, then the value of $x^7 + (1/x^5)$ is:

(a) 2^5 (b) 2^{12} (c) 2 (d) 2^7

(SSS CPO 21–06–2015 MORNING)

324. The term, that should be added to $(4x^2 + 8x)$ so that resulting expression be a perfect square, is:

(a) $2x$ (b) 2 (c) 1 (d) 4

(SSS CPO 21–06–2015 MORNING)

325. If 999x + 888y = 1332 and 888x + 999y = 555

Then the value of x + y is?

(a) 888 (b) 1
(c) 555 (d) 999

(SSS CPO 21–06–2015 MORNING)

326. If $a^2 + b^2 + c^2 = ab + bc + ca$, then the value of $\frac{a+c}{b}$ is

(a) 0 (b) 2 (c) 1 (d) 3

(SSS CPO 21–06–2015 EVENING)

327. If $x = \frac{1}{2+\sqrt{3}}$, $y = \frac{1}{2-\sqrt{3}}$, then the value of $8xy(x^2 + y^2)$ is

(a) 112 (b) 194
(c) 290 (d) 196

(SSS CPO 21–06–2015 EVENING)

328. If $a = \frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$, then the value of $a^2 - ax$ is

(a) 2 (b) 1 (c) 0 (d) – 1

(SSS CPO 21–06–2015 EVENING) MAINS 2015

329. If a + b = 1, find the value of $a^3 + b^3 - ab - (a^2 - b^2)^2$

(a)0 (b)1 (c) – 1 (d)2

(CGL mains 25–10–2015)

330. If $a - \frac{1}{a-3} = 5$, then the value of $(a-3)^3 - \frac{1}{(a-3)^3}$

(a) 7 (b) 14 (c) 2 (d) 5

(CGL mains 25–10–2015)

331. $(3x-2y) : (2x+3y) = 5 : 6$, then one of the value of $\left(\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right)^2$ is

(a) $\frac{1}{25}$ (b) 5 (c) $\frac{1}{5}$ (d) 25

(CGL mains 25–10–2015)

332. If $x - \sqrt{3} - \sqrt{2} = 0$ and $y - \sqrt{3} + \sqrt{2} = 0$, then value of $(x^3 - 20\sqrt{2}) - (y^3 + 2\sqrt{2})$

(a) 2 (b) 3 (c) 1 (d) 0

(CGL mains 25–10–2015)

333. $3(a^2+b^2+c^2) = (a+b+c)^2$, then the relation between a, b and c is

(a) a = b ≠ c (b) a ≠ b ≠ c
(c) a ≠ b = c (d) a = b = c

(CGL mains 25–10–2015)

334. $x = a^{\frac{1}{2}} + a^{-\frac{1}{2}}$, $y = a^{\frac{1}{2}} - a^{-\frac{1}{2}}$, then value of

$(x^4 - x^2y^2 - 1) + (y^4 - x^2y^2 + 1)$

(a) 13 (b) 12 (c) 14 (d) 16

(CGL mains 25–10–2015)

335. If $m = \sqrt{5+\sqrt{5+\sqrt{5.....}}}$

$n = \sqrt{5-\sqrt{5-\sqrt{5-.....}}}$

then among the following the relation between m & n holds is

(a) m–n+1=0 (b) m+n+1=0
(c) m+n–1=0 (d) m–n–1=0

(CGL mains 25–10–2014)

336. If $\frac{3-5x}{2x}+\frac{3-5y}{2y}+\frac{3-5z}{2z}=0$, then the value of $\frac{2}{x}+\frac{2}{y}+\frac{2}{z}$ is

(a) 20 (b) 10 (c) 5 (d) 15

(CGL mains 25–10–2014)

337. If 2s = a + b + c, then the value of s(s – c) + (s –a) (s – b) is

(a) ab (b) 0

(c) abc (d) $\frac{a+b+c}{2}$

(CGL mains 25–10–2014)

338. If p + m = 6 and $p^3 + m^3 = 72$, then the value of pm is

(a) 6 (b) 9 (c) 12 (d) 8

(CGL mains 25–10–2014)

339. When x^m is multiplied by x^n, product is 1. The relation between m and n is

(a) mn = 1 (b) m + n = 1

(c) m = n (d) m = – n

(CGL mains 25–10–2014)

340. The area (in sq. unit) of the triangle formed by the graphs of the equations $x = 4$, y = 3 and $3x + 4y = 12$; is

(a) 24 (b) 6 (c) 12 (d) 3

(CGL mains 25–10–2014)

341. If the ordinate and abscissa of the point (K, 2K–1) be equal, then the value of K is

(a) 0 (b) 1 (c) – 1 (d) $\frac{1}{2}$

342. If $x = 5$, then the value of the expression $x^2-2+\frac{1}{x^2}$ is

(a) $\frac{576}{25}$ (b) $\frac{24}{25}$ (c) $\frac{24}{5}$ (d) $\frac{625}{24}$

(LDC 01–11–2015 MORNING)

343. If $x = 2$, y = 1 and z = – 3, then $x^3 + y^3 + z^3 - 3xyz$ is equal to

(a) 6 (b) 0 (c) 2 (d) 8

(LDC 01–11–2015 MORNING)

344. If $a+\frac{1}{a} = -1$, then the value of $(1 - a + a^2)(1 + a - a^2)$ is

(a) 1 (b) 0 (c) – 4 (d) 4

(LDC 01–11–2015 MORNING)

345. $(x^3 + y^6)(x^3 - y^6)$ is equal to

(a) $x^6 - y^{12}$ (b) $x^9 - y^{16}$

(c) $x^6 + y^{12}$ (d) $x^9 + y^{36}$

(LDC 01–11–2015 MORNING)

346. The sum of $\frac{1}{x+y}$ and $\frac{1}{x-y}$ is

(a) $\frac{2y}{x^2-y^2}$ (b) $\frac{2x}{x^2-y^2}$

(c) $\frac{-2y}{x^2-y^2}$ (d) $\frac{2x}{y^2-x^2}$

(LDC 01–11–2015 MORNING)

347. If $x + y = 2a$, then the value of $\frac{a}{x-a}+\frac{a}{y-a}$ is

(a) 0 (b) – 1 (c) 1 (d) 2

(LDC 01–11–2015 EVENING)

348. If $p^3 - q^3 = (p - q)\{(p - q)^2 - xpq\}$, then find the value of x is:

(a) – 1 (b) 3 (c) 1 (d) – 3

(LDC 01–11–2015 EVENING)

349. If $x + y + z = 6$ and $xy + yz + zx = 10$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is:

(a) 36 (b) 40 (c) 42 (d) 48

(LDC 15–11–2015 MORNING)

350. If $\frac{x+1}{x-1}=\frac{a}{b}$ and $\frac{1-y}{1+y}=\frac{b}{a}$, then the value of $\frac{x-y}{1+xy}$ is:

(a) $\frac{a^2-b^2}{ab}$ (b) $\frac{a^2+b^2}{2ab}$

(c) $\frac{a^2-b^2}{2ab}$ (d) $\frac{2ab}{a^2-b^2}$

(LDC 15–11–2015 MORNING)

351. If $a^2 + a + 1 = 0$, then the value of $a^5 + a^4 + 1$ is:

(a) 1 (b) 0 (c) a + 1 (d) a^2

(LDC 15–11–2015 MORNING)

352. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$ then a : b : c is:

(a) 1 : 2 : 1 (b) 2 : 1 : 1

(c) 1 : 1 : 2 (d) 1 : 1 : 1

(LDC 15–11–2015 MORNING)

353. If $x-\frac{1}{x}=2$, then the value of the following is $x^3-\frac{1}{x^3}=?$

(a) 2 (b) 11 (c) 15 (d) 14

(LDC 15–11–2015 MORNING)

354. If $x = a(b - c)$, $y = b(c - a)$, $z = c(a - b)$ then the value of $\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3+\left(\frac{z}{c}\right)^3$ is:

(a) $\frac{xyz}{abc}$ (b) 0

(c) $\frac{3xyz}{abc}$ (d) $\frac{2xyz}{abc}$

(LDC 15–11–2015 EVENING)

355. If $\sqrt{y}=4x$, then $\frac{x^2}{y}$ is:

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$ (c) 4 (d) 2

(LDC 15–11–2015 EVENING)

356. If $\frac{x}{y}=\frac{a+2}{a-2}$, then the value of $\frac{x^2-y^2}{x^2+y^2}$

(a) $\frac{2a}{a^2+2}$ (b) $\frac{4a}{a^2+4}$

(c) $\frac{2a}{a^2+4}$ (d) $\frac{4a}{a^2+2}$

(LDC 15–11–2015 EVENING)

357. If $x = y = z$, then $\frac{(x+y+z)^2}{x^2+y^2+z^2}$ is:

(a) 2 (b) 3 (c) 1 (d) 4

(LDC 15–11–2015 EVENING)

358 If $\frac{a}{b}+\frac{b}{a}=2$, then the value of a – b is:

(a) 2 (b) –1 (c) 0 (d) 1

(LDC 15–11–2015 EVENING)

359. If $x(x + y + z)=20$, $y(x + y + z) = 30$, & $z(x + y + z) = 50$, then the value of $2(x + y + z)$ is:

(a) 20 (b) 10 (c) 15 (d) 18

(LDC 6–12–2015 MORNING)

360. If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then the value of a + b + c is:

(a) –2 (b) 1

(c) 2 (d) –1

(LDC 6–12–2015 MORNING)

361. If $x + y = 4$, $x^2 + y^2 = 14$ and $x > y$. Then the correct value of x and y is:

(a) $2-\sqrt{2}, \sqrt{3}$ (b) 3, 1

(c) $2+\sqrt{3}, 2-\sqrt{3}$ (d) $2+\sqrt{3}, 2\sqrt{2}$

(LDC 6–12–2015 MORNING)

362. If for non-zero x, $x^2 - 4x - 1 = 0$ the value of is $x^2 + \frac{1}{x^2}$:

(a) 4 (b) 10 (c) 12 (d) 18

(LDC 6–12–2015 EVENING)

363. The simplified value of following is:

$$\left(\frac{3}{15}a^5 b^6 c^3 \times \frac{5}{9}a b^5 c^4\right) \div \frac{10}{17}a^2 b c^3$$

(a) $\frac{3}{10}a b^4 c^3$ (b) $\frac{9}{10}a^2 b c^4$

(c) $\frac{3}{10}a^4 b^{10} c^4$ (d) $\frac{1}{10}a^4 b^4 c^{10}$

(LDC 6–12–2015 EVENING)

364. If $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then $\frac{a^2}{b} + \frac{b^2}{a}$ value of :

(a) 1030 (b) 970

(c) 1025 (d) 930

(LDC 6–12–2015 EVENING)

365. If $(2a - 1)^2 + (4b - 3)^2 + (4c + 5)^2 = 0$ then the value of $\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2}$ is:

(a) $3\frac{3}{8}$ (b) $2\frac{3}{8}$ (c) 0 (d) $1\frac{3}{8}$

(LDC 6–12–2015 EVENING)

366. If $\frac{a}{b} = \frac{25}{6}$, then the value of $\frac{a^2 - b^2}{a^2 + b^2}$ is

(a) $\frac{589}{651}$ (b) $\frac{589}{661}$ (c) $\frac{661}{589}$ (d) $\frac{625}{36}$

(LDC 20–12–2015 MORNING)

367. If (2, 0) is a solution of the linear equation $2x + 3y = K$, then the value of K is

(a) 4 (b) 6 (c) 5 (d) 2

(LDC 20–12–2015 MORNING)

368. If $x + \frac{1}{x} = 3$ then the value of $x^5 + \frac{1}{x^5}$ is

(a) 123 (b) 110

(c) 132 (d) 122

(LDC 20–12–2015 MORNING)

369. The graph of linear equation $y = x$ passes throughout the point

(a) $\left(\frac{3}{2}, -\frac{3}{2}\right)$ (b) $\left(0, -\frac{3}{2}\right)$

(c) (1, 1) (d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(LDC 20–12–2015 MORNING)

370. If $\left(a + \frac{1}{a}\right)^2 = 3$, then find the value of $a^{30} + a^{24} + a^{18} + a^{12} + a^6 + 1$

(a) 0 (b) 27 (c) 1 (d) –1

(LDC 20–12–2015 EVENING)

371. If $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$, then the value of $a^3 - b^3$ is:

(a) 3 (b) 2 (c) 1 (d) 0

(LDC 20–12–2015 EVENING)

372. If $x : y = 3:5$ and $x - y = -2$, then the value of $x+y$ is:

(a) 8 (b) 2 (c) 3 (d) 5

(LDC 20–12–2015 EVENING)

373. If $x + 1 = \sqrt{y} + 3, y > 0$, then the value of $\frac{1}{2}\left(\frac{x^3 - 6x^2 + 12x - 8}{\sqrt{y}} - y\right)$ is.

(a) $\frac{1}{2}$ (b) –1 (c) 1 (d) 0

(SSC CPO 20-03-2016, Morning)

374. If $x = 1 + \sqrt{2} + \sqrt{3}$ and $y = 1 + \sqrt{2} - \sqrt{3}$, then the value of $\frac{x^2 + 4xy + y^2}{x + y}$ is

(a) $2\sqrt{2}$ (b) $2(2 + \sqrt{2})$

(c) 1 (d) 6

(SSC CPO 20-03-2016, Morning)

375. If $x + \frac{1}{x} = 3$, where $x \neq 0$, then the value of

$$\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$$

(a) 3 (b) 5 (c) 7 (d) 2

(SSC CPO 20-03-2016, Morning)

376. If $x + \frac{1}{x} = -2$, then the value of $x^p + x^q$ is:

(where P is an even number and q is an odd number)

(a) – 2 (b) 1 (c) 0 (d) 2

(SSC CPO 20-03-2016, Evening)

377. If $P(x + y)^2 = 5$ and $q(x - y)^2 = 3$ then the simplified value of $p^2(x + y)^2 + 4pq\,xy - q^2(x - y)^2$ is:

(a) 2 (p +q) (b) – (p + q)

(c) – 2(p +q) (d) p + q

(SSC CPO 20-03-2016, Evening)

378. If p,q, r are all real numbers, then $(p - q)^3 + (q - r)^3 + (r - p)^3$ is equal to

(a) 3(p – q)(q – r) (r –P)

(b) 1 (c) 0

(d) (p – q)(q – r) (r – p)

(SSC CPO 20-03-2016, Morning)

379. If $a + \frac{1}{a} = \sqrt{3}$ then the value of $a^{52} + \frac{1}{a^{52}}$ is:

(a) –1 (b) 0 (c) 1 (d) 2

(SSC CPO(Re) 04-06-2016, Morning)

380. If σ and β are the roots of equation $x^2 + \sigma x + \beta = 0$ then find $\sigma^3 + \beta^3$

(a) –7 (b) 8 (c) –8 (d) 7

(SSC CPO(Re) 04-06-2016, Morning)

381. If $x^2 + \frac{1}{x^2} = 1$ then the value of $x^{102} + x^{96} + x^{90} + x^{84} + x^{78} + x^{72} + 5$ is

(a) 0 (b) 5

(c) 3 (d) 1

(SSC CPO(Re) 04-06-2016, Morning)

382. Find the value of a and b if $(x - 1)$ and $(x + 1)$ are factors of $x^4 + ax^3 - 3x^2 + 2x + b$

(a) 2, –1 (b) –2, 1

(c) –2, 2 (d) 1, –1

(SSC CPO(Re) 04-06-2016, Morning)

383. Find the minimum value of x which the expression $x^3 - 7x^2 + 11x - 5 \geq 0$.

(a) 0 (b) 5 (c) 1 (d) –3

(SSC CPO(Re) 04-06-2016, Evening)

384. If a + b + c = 26 and ab + bc + ca = 109, find the value of $a^2 + b^2 + c^2$

(a) 458 (b) 472 (c) 452 (d) 476

(SSC CPO(Re) 04-06-2016, Evening)

385. If $x^2 + \frac{1}{x^2} = 1$, then the value of $x^{18} + x^{12} + x^6 + 1$ is:

(a) 1 (b) 2 (c) 0 (d) –1

(SSC CPO(Re) 04-06-2016, Evening)

386. a + b + c = 0 then the value of $\frac{a^2+b^2+c^2}{ab+bc+ca}$ is

(a) 2 (b) – 2 (c) 0 (c) 4

(SSC CPO(Re) 05-06-2016, Morning)

387. If a + b + c = m and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ = 0 then average of a^2, b^2, c^2 is

(a) m^2 (b) $m^2/3$

(c) $m^2/9$ (d) $m^2/27$

(SSC CPO(Re) 05-06-2016, Morning)

388. The value of $(2a + b)^2 - (2a - b)^2$ is:

(a) 8ab (b) – 8ab

(c) $8a^2 + 2b^2$ (d) $8a^2 - 2b^2$

(SSC CPO(Re) 05-06-2016, Morning)

389. If $x = \frac{8ab}{a+b} (a \neq b)$ then the value of $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$ is:

(a) 0 (b) 1 (c) 2 (d) 4

(SSC CPO(Re) 05-06-2016, Evening)

390. If $(\sqrt{a}+\sqrt{b}) = 15$ and $(\sqrt{a}-\sqrt{b})$ = 3 then the value of $\sqrt{ab}/4$ is

(a) 6 (b) 7

(c) 27/2 (d) 5

(SSC CPO(Re) 05-06-2016, Evening)

391. If $a + \frac{1}{a} = 3$ then the value of $a^3 + \frac{1}{a^3}$ is

(a) 27 (b) 24 (c) 19 (d) 18

(SSC CPO(Re) 05-06-2016, Evening)

392. If a – 6b = 3 then the value of $a^3 - 216b^3 - 54ab$ is

(a) 9 (b) 1 (c) 27 (d) 64

(SSC CPO(Re) 06-06-2016, Morning)

393. If $x = (10 + 3\sqrt{11})$, What is the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$?

(a) $2\sqrt{3}$ (b) $\sqrt{22}$

(c) 18 (d) $3\sqrt{2}$

(SSC CPO(Re) 06-06-2016, Morning)

394. If $x = \sqrt{a\sqrt[3]{ab\sqrt{a\sqrt[3]{ab}}}}$ .....∞, then the value of x is:

(a) $\sqrt[5]{a^2b}$ (b) $\sqrt[5]{a^4b^4}$

(c) $\sqrt[6]{a^5b}$ (d) $\sqrt[5]{a^4b}$

(SSC CPO(Re) 06-06-2016, Morning)

395. If $\frac{m-3a^3}{b^3+c^3} + \frac{m-3b^3}{c^3+a^3} + \frac{m-3c^3}{a^3+b^3}$ = 9, then the value of m is:

(a) $a^3 + b^3 + c^3$

(b) $2a^3 + 2b^3 + 2c^3$

(c) $3a^3 + 3b^3 + 3c^3$

(d) 2

(SSC CPO(Re) 06-06-2016, Evening)

396. If the polynomial $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, Find the value of a.

(a) 1 (b) –1 (c) 2 (d) –2

(SSC CPO(Re) 06-06-2016, Evening)

397. If $(2a - 3)^2 + (3b + 4)^2 + (6c + 1)^2 = 0$, then the value of $\frac{a^3+b^3+c^3-3abc}{a^3+b^3+c^3} + 3$ is:

(a) abc + 3 (b) 6

(c) 0 (d) 3

(SSC CPO(Re) 06-06-2016, Evening)

398. If $a^2 = b + c$, $b^2 = a + c$, $c^2 = b + a$, then what will be the value of $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1}$?

(SSC CPO(Re) 07-06-2016, Morning)

(a) –1 (b) 2 (c) 1 (d) 0

399. If a + b = 2c, find $\frac{a}{a-c} + \frac{c}{b-c}$

(a) 0 (b) 1 (c) 2 (d) –1

(SSC CPO(Re) 07-06-2016, Morning)

400. What will be the value of $x^3 + y^3 + z^3 - 3xyz$ when $x + y + z = 9$ and $x^2 + y^2 + z^2 = 31$?

(a) 27 (b) 3 (c) 54 (d) 9

(SSC CPO(Re) 07-06-2016, Morning)

401. If a + b + c = 1, ab + bc + ca = –1 and abc = –1, then the value of $a^3 + b^3 + c^3$ is:

(a) 1 (b) –1 (c) 2 (d) –2

(SSC CPO(Re) 08-06-2016, Morning)

402. If $\frac{a}{b} = \frac{1}{2}$, find the value of the expression $\frac{(2a-5b)}{(5a+3b)}$

(a) –32 (b) 11 (c) $\frac{-8}{11}$ (d) 17

(SSC CPO(Re) 08-06-2016, Evening)

403. If for a non-zero x, $3x^2 + 5x + 3 = 0$, then the value of $x^3 + \frac{1}{x^3}$ is:

(a) $\frac{10}{27}$ (b) $-\left(\frac{10}{27}\right)$

(c) $\frac{2}{3}$ (d) $-\left(\frac{2}{3}\right)$

(SSC CPO(Re) 08-06-2016, Evening)

404. If a/b + b/a = 1, then the value of $a^3 + b^3$ will be

(a) 1 (b) 0 (c) – 1 (d) 2

(SSC CGL Pre Exam 2016)

405. If p = 99, then the value of $p(p^2 + 3p + 3)$ will be

(a) 999999 (b) 1000000

(c) 1000001 (d) 999998

(SSC CGL Pre Exam 2016)

406. If a – b = 1 and $a^3 - b^3 = 61$, then the value of ab will be

(a) –20 (b) 20 (c) 30 (d) 60

(SSC CGL Pre Exam 2016)

407. If $\frac{x}{a+b} + 1 = \frac{x}{a-b} + \frac{a-b}{a+b}$, then x is equal to

(a) 2a – b (b) a + b

(c) a – b (d) 2a + b

(SSC CGL Pre Exam 2016)

408. If $x^2 + 5x + 6 = 0$, then the value of $\frac{2x}{x^2-7x+6}$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $-\frac{1}{6}$ (d) $-\frac{1}{3}$

(SSC CGL Pre Exam 2016)

409. If a + b = 5 and a – b = 3, then the value of $(a^2 + b^2)$ is

(a) 17 (b) 18 (c) 19 (d) 20

(SSC CGL Pre Exam 2016)

410. If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$, then the value of $x4 + y4 - 2x^2y^2$ is

(a) 4 (b) 8 (c) 16 (d) 64

(SSC CGL Pre Exam 2016)

411. If $2^{x+4} - 2^{x+2} = 3$, then the value of 'x' is

(a) 0 (b) 2 (c) –1 (d) –2

(SSC CGL Pre Exam 2016)

412. If $x + \frac{1}{x} = \sqrt{3}$, then find the value of $x^3 + \frac{1}{x^3}$

(a) –2 (b) 0 (c) 2 (d) 4

(SSC CGL Pre Exam 2016)

413. If $a + \frac{1}{a} = 2$, then find the value of $a^{109} + \frac{1}{a^{109}}$

(a) 1 (b) 2 (c) 0 (d) 109

(SSC CGL Pre Exam 2016)

414. If $2x - \frac{1}{2x} = 5$, $x \neq 0$, then find the value of $x^2 + \frac{1}{16x^2} - 2$

(a) $\frac{19}{4}$ (b) $\frac{23}{4}$ (c) $\frac{27}{4}$ (d) $\frac{31}{4}$

(SSC CGL Pre Exam 2016)

415. If $x = 1/\left(\sqrt{2}+1\right)$ Then the value of $x^2 + 2x - 1$ is

(a) $2\sqrt{2}$ (b) 4 (c) 0 (d) 2

(SSC CGL Pre Exam 2016)

416. If $x + 1/x = \sqrt{13}$, then $3x/(x^2-1)$ equal to

(a) $3\sqrt{13}$ (b) $\frac{\sqrt{13}}{13}$

(c) 1 (d) 3

(SSC CGL Pre Exam 2016)

417. If $a(x+y) = b(x-y) = 2ab$, then the value of $2(x^2+y^2)$ is

(a) $2(a^2 - b^2)$ (b) $2(a^2+b^2)$

(c) $4(a^2 - b^2)$ (d) $4(a^2 + b^2)$

(SSC CGL Pre Exam 2016)

418. If $x + 1/x = 6$, then value of $x^2 + 1/x^2$ is

(a) 23 (b) 16 (c) 34 (d) 32

(SSC CGL Pre Exam 2016)

419. If $C + \frac{1}{c} = 3$, then value of $(C-3)^7 + \frac{1}{c^7}$ is

(a) 2 (b) 0 (c) 3 (d) 1

(SSC CGL Pre Exam 2016)

420. If $2x + \frac{1}{4x} = 1$ Then value of $x^2 + \frac{1}{64x^2}$ is

(a) 0 (b) 1 (c) 1/4 (d) 2

(SSC CGL Pre Exam 2016)

421. If $\sqrt{x} - \sqrt{y} = 1$, $\sqrt{x} + \sqrt{y} = 17$ then $\sqrt{xy}$ = ?

(a) $\sqrt{72}$ (b) 72

(c) 32 (d) 24

(SSC CGL Pre Exam 2016)

422. If $x = \sqrt{3} + \frac{1}{\sqrt{3}}$, then the value of

$$\left(x - \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x - \frac{1}{x - \frac{2\sqrt{3}}{3}}\right) \text{ is}$$

(a) $5\frac{\sqrt{3}}{6}$ (b) $\frac{2\sqrt{3}}{3}$

(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

(SSC CGL Pre Exam 2016)

423. If $a^3 - b^3 = 56$ and $a - b = 2$ then what is the value of $a^2 + b^2$?

(a) 12 (b) 20 (c) 28 (d) 32

(SSC CGL Pre Exam 2016)

424. If $x^2 - 3x + 1 = 0$, $(x \neq 0)$, then the value of $x + \frac{1}{x}$

(a) 1 (b) 0 (c) 3 (d) 2

(SSC CGL Pre Exam 2016)

425. If $\frac{2+a}{a} + \frac{2+b}{b} + \frac{2+c}{c} = 4$, then the value of $\frac{ab+bc+ca}{abc}$ is

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$

(SSC CGL Pre Exam 2016)

426. If $x + y + z = 1$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ and $xyz = -1$, then $x^3 + y^3 + z^3$ equal to

(a) –1 (b) 1 (c) –2 (d) 2

(SSC CGL Pre Exam 2016)

427. If $x + \frac{1}{x} = 5$, then the value of $\frac{5x}{x^2 + 5x + 1}$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

(SSC CGL Pre Exam 2016)

428. If $\frac{1}{a}(a^2 + 1) = 3$, then the value of $\frac{a^6 + 1}{a^3}$

(a) 9 (b) 18 (c) 27 (d) 1

(SSC CGL Pre Exam 2016)

429. If $a = \sqrt{2} + 1$, $b = \sqrt{2} - 1$, then the value of $\frac{1}{a+1} + \frac{1}{b+1}$ is

(a) 0 (b) 1 (c) 2 (d) 3

(SSC CGL Pre Exam 2016)

430. If $(x-5)^2 + (y-2)^2 + (z-9)^2 = 0$, then value of $(x + y - z)$ is

(a) 16 (b) –1 (c) –2 (d) 12

(SSC CGL Pre Exam 2016)

431. If $x + \frac{1}{x} = 3$ then $x^8 + \frac{1}{x^8}$ is equal to

(a) 2201 (b) 2203

(c) 2207 (d) 2213

(SSC CGL Pre Exam 2016)

432. If $x = 999$, $y = 1000$, $z = 1001$, then the value of

$$\frac{x^3 + y^3 + z^3 - 3xyz}{x - y + z} \text{ is}$$

(a) 1000 (b) 9000

(c) 1 (d) 9

(SSC CGL Pre Exam 2016)

433. If $\frac{a}{1-2a} + \frac{b}{1-2b} + \frac{c}{1-2c} = \frac{1}{2}$, Then the value $\frac{1}{1-2a} + \frac{1}{1-2b} + \frac{1}{1-2c}$ is?

(a) 1 (b) 2 (c) 3 (d) 4

(SSC CGL Pre Exam 2016)

434. Ratio of three numbers x,y,z are in 2, 3, 5 respectively and the sum of x, y and z is 80. If the number z is given by the equation z = ax – 8, then a is

(a) 6 (b) 3/2
(c) 3 (d) 5/2

(SSC CGL Pre Exam 2016)

435. If (x – 2) (x – p) = x^2 – ax + 6, then the value of (a – p) is

(a) 0 (b) 1
(c) 2 (d) 3

(SSC CGL Pre Exam 2016)

436. If $x = \sqrt{a} + \frac{1}{\sqrt{a}}$, $y = \sqrt{a} - \frac{1}{\sqrt{a}}$, (a > 0), then the value of $x^4 + y^4 - 2x^2y^2$ is

(a) 16 (b) 20
(c) 10 (d) 5

(SSC CGL Pre Exam 2016)

437. If $x = \sqrt[3]{x^2+11} - 2$, then the value of $x^3 + 5x^2 + 12x$ is

(a) 0 (b) 3
(c) 7 (d) 11

(SSC CGL Pre Exam 2016)

438. If $P^2 + \frac{1}{p^2} = 47$, the value of $P + \frac{1}{p}$ is

(a) 5 (b) 6
(c) 7 (d) 8

(SSC CGL Pre Exam 2016)

439. The third proportional of the following numbers $(x - y)^2$, $(x^2-y^2)^2$

(a) $(x + y^3)(x + y^4)$
(b) $(x + y)^4 (x - y)^2$
(c) $(x + y)(x - y)^2$
(d) $(x + y)^2 (x - y)^3$

(SSC CGL Pre Exam 2016)

440. If $x + \sqrt{5} = 5 + \sqrt{y}$ and x, y are positive integers, then the value of $\frac{\sqrt{x}+y}{x+\sqrt{y}}$ is

(a) 1 (b) 2
(c) $\sqrt{5}$ (d) 5

(SSC CGL Pre Exam 2016)

441. If x, y and z are real numbers such that $(x - 3)^2 + (y - 4)^2 + (z - 5)^2 = 0$ then $(x+y+z)$ is equal to

(a) 10 (b) 14
(c) 11 (d) 12

(SSC CGL Pre Exam 2016)

442. If $2x + \frac{1}{3x} = 5$, the value of $\frac{5x}{6x^2+20x+1}$ is

(a) 1/4 (b) 1/6
(c) 1/5 (d) 1/7

(SSC CGL Pre Exam 2016)

443. If $a + b = 10$ and $ab = 21$, then the value of $(a - b)^2$ is

(a) 15 (b) 16
(c) 17 (d) 18

(SSC CGL Pre Exam 2016)

444. If $(x - 4)(x^2 + 4x + 16) = x^3 - P$, then P is equal to

(a) 27 (b) 8
(c) 64 (d) 0

(SSC CGL Pre Exam 2016)

445. If $4x + \frac{1}{x} = 5$, $x \neq 0$, then the value of $\frac{5x}{4x^2+10x+1}$ is

(a) 1/2 (b) 1/3
(c) 2/3 (d) 3

(SSC CGL Pre Exam 2016)

446. If $C + \frac{1}{C} = \sqrt{3}$, then the value of $C^3 + \frac{1}{C^3}$ is equal to

(a) 0 (b) $\frac{3}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{6}{\sqrt{3}}$

(SSC CGL Pre Exam 2016)

447. If $x = 222$, $y=223$, $z=225$ then the value of $x^3+y^3+z^3-3xyz$ is

(a) 4590 (b) 4690
(c) 4950 (d) 4960

(SSC CGL Pre Exam 2016)

448. If a + b + c = 0, then the value of $a^3 + b^3 + c^3$ is

(a) abc (b) 2abc
(c) 3abc (d) 0

(SSC CGL Pre Exam 2016)

449. The value of $\left(x^{\frac{1}{3}} + x^{\frac{-1}{3}}\right)\left(x^{\frac{2}{3}} - 1 + x^{\frac{-2}{3}}\right)$ is

(a) $x^{-1} + x^{\frac{2}{3}}$ (b) $x + x^{-\frac{1}{3}}$
(c) $x^{\frac{1}{3}} + x^{-1}$ (d) $x + x^{-1}$

(SSC CGL Pre Exam 2016)

450. If, $\frac{1}{p} + \frac{1}{q} = \frac{1}{p+q}$, then the value of $p^3 - q^3$ is

(a) p – q (b) p q
(c) 1 (d) 0

(SSC CGL Pre Exam 2016)

451. If $x = 93$, $y = 93$, $z = 94$ then the value of $x^2 - y^2 + 10xz + 10yz$ is

(a) 104784 (b) 147840
(c) 174840 (d) 184740

(SSC CGL Pre Exam 2016)

452. If ab = 21 and $\frac{(a+b)^2}{(a-b)^2} = \frac{25}{4}$ then the value of $a^2 + b^2 + 3ab$ is

(a) 115 (b) 121
(c) 125 (d) 127

(SSC CGL Pre Exam 2016)

453. Given $a - b = 2$, $a^3 - b^3 = 26$ then $(a + b)^2$ is

(a) 9 (b) 4
(c) 16 (d) 12

(SSC CGL Pre Exam 2016)

454. If $x + y + z = 9$ then the value of $(x - 4)^3 + (y - 2)^3 + (z - 3)^3 - 3(x - 4)(y-2)(z - 3)$ is

(a) 6 (b) 9
(c) 0 (d) 1

(SSC CGL Pre Exam 2016)

455. If $x + \frac{1}{9x} = 4$, then $9x^2 + \frac{1}{9x^2}$ is

(a) 140 (b) 142
(c) 144 (d) 146

(SSC CGL Pre Exam 2016)

456. If a = 2, b = – 3 then the value of $27a^3 - 54a^2b + 36ab^2 - 8b^3$ is

(a) 1562 (b) 1616
(c) 1676 (d) 1728

(SSC CGL Pre Exam 2016)

457. If $a^3 + \frac{1}{a^3} = 2$, then value of $\frac{a^2+1}{a}$ is (a is a positive number)

(a) 1 (b) 2 (c) 3 (d) 4

(SSC CGL Pre Exam 2016)

458. If $x\left(3-\frac{2}{x}\right)=\frac{3}{x}$, then the value of $x^2+\frac{1}{x^2}$ will be

(a) $3\frac{1}{9}$ (b) $3\frac{2}{9}$ (c) $2\frac{1}{9}$ (d) $2\frac{4}{9}$

(SSC CGL Pre Exam 2016)

459 If $\frac{a}{q-r}=\frac{b}{r-p}=\frac{c}{p-q}$, find the value of pa + qb + rc is

(a) 0 (b) 21
(c) 2 (d) –

(SSC CGL Pre Exam 2016)

460. If $\left(a+\frac{1}{a}\right)^2=3$, the value of $a^3+\frac{1}{a^3}$

(a) 0 (b) $3\left(a+\frac{1}{a}\right)$

(c) $3\left(a^2+\frac{1}{a^2}\right)$ (d) 1

(SSC CGL Pre Exam 2016)

461. If $\frac{a^2+b^2}{c^2}=\frac{b^2+c^2}{a^2}=\frac{c^2+a^2}{b^2}=\frac{1}{k}$, $(k\neq 0)$ then k =

(a) 2 (b) 1
(c) 0 (d) 1/2

(SSC CGL Pre Exam 2016)

462. If $2x+\frac{2}{9x}=4$, then the value of $27x^3+\frac{1}{27x^3}$ is

(a) 180 (b) 198
(c) 234 (d) 252

(SSC CGL Pre Exam 2016)

463. If $x^2+1/x^2=2$, then the value of $x-1/x$ is

(a) –2 (b) 0
(c) 1 (d) –1

(SSC CGL Pre Exam 2016)

464. If pq (p + q) = 1, Then the value of $\frac{1}{P^3q^3}-p^3-q^3$ is equal to

(a) 1 (b) 2
(c) 3 (d) 4

(SSC CGL Pre Exam 2016)

465. If $x=\sqrt{2}+1$, then the value of x^4-1/x^4 is

(a) $8\sqrt{2}$ (b) $18\sqrt{2}$

(c) $6\sqrt{2}$ (d) $24\sqrt{2}$

(SSC CGL Pre Exam 2016)

466. If $p^3-q^3=(p-q)\{(p+q)^2-xpq\}$ then the value of x is

(a) 1 (b) – 1
(c) 2 (d) – 2

(SSC CGL Pre Exam 2016)

467. If $x=(0.25)^{1/2}$, $y=(0.4)^2$, $z=(0.216)^{1/2}$ then

(a) Y > X > Z (b) X > Y > Z
(c) Z > X > Y (d) X > Z > Y

(SSC CGL Pre Exam 2016)

468. If $a^2=by+cz$, $b^2=cz+ax$, $c^2=ax+by$, then the value of $\frac{x}{a+x}+\frac{y}{b+y}+\frac{z}{c+z}$ is

(a) 1 (b) a+b+c

(c) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ (d) 0

(SSC CGL Pre Exam 2016)

469. If $x^2+y^2=29$ and xy = 10, where $x>0$, $y>0$, $x>y$ then the value of $\frac{x+y}{x-y}$ is

(a) –7/3 (b) 7/3
(c) 3/7 (d) –3/7

(SSC CGL Pre Exam 2016)

470. If $\frac{a}{b}+\frac{b}{a}=1$, Then are value of a^3+b^3 is equal to

(a) 0 (b) 1
(c) 2 (d) 3

(SSC CGL Pre Exam 2016)

471. if $p^2+q^2=7pq$, Then the value of $\frac{p}{q}+\frac{q}{p}$ is equal to

(a) 9 (b) 5
(c) 7 (d) 3

(SSC CGL Pre Exam 2016)

472. If $x=3^{\frac{1}{3}}-3^{-\frac{1}{3}}$ value of $3x^3+9x$ is

(a) 8 (b) 9
(c) 27 (d) 16

(SSC CGL Pre Exam 2016)

473. If $xy(x+y)=m$, then the value of x^3+y^3+3m is:

(a) m^3/xy (b) $m^3(x+y)^2$
(c) m^3/x^3y^3 (d) mx^3y^3

(SSC CGL Pre Exam 2016)

474. If $P+\frac{1}{p+2}=1$, then the value of $(p+2)^3+\frac{1}{(p+2)^3}-3$ is

(a) 12 (b) 16
(c) 18 (d) 15

(SSC CGL Pre Exam 2016)

475. If $x+\frac{1}{x}\neq 0$ and $x^3+\frac{1}{x^3}=0$ then the value $\left(x+\frac{1}{x}\right)^4$ is

(a) 9 (b) 12
(c) 15 (d) 16

(SSC CGL Pre Exam 2016)

476. If $x+\frac{1}{x}=2$ then $x^2+\frac{1}{x^2}$ is equal to

(a) 0 (b) 2
(c) 4 (d) 8

(SSC CGL Pre Exam 2016)

477. If $x+y=\sqrt{3}$ and $x-y=\sqrt{2}$, then the value of $8xy(x^2+y^2)$ is

(a) 6 (b) $\sqrt{6}$

(c) 5 (d) $\sqrt{5}$

(SSC CGL Pre Exam 2016)

478. If $\left(x+\frac{1}{x}\right):\left(x-\frac{1}{x}\right)=5:3$, the value of x is/are

(a) ± 1 (b) ± 2
(c) ± 3 (d) 0

(SSC CGL Pre Exam 2016)

479. If $p^3-q^3=(p-q)[(p-q)^2+x\,pq)]$ then value of x is

(a) 1 (b) –1
(c) 3 (d) 2

(SSC CGL Pre Exam 2016)

480. If $\left(a+\frac{1}{a}\right)^2=3$, then the value of $a^{18}+a^{12}+a^6+1$ is

(a) 3 (b) 1
(c) 0 (d) 2

(SSC CGL Pre Exam 2016)

481. $(113^2 + 115^2 + 117^2 - 113 \times 115 - 115 \times 117 - 113 \times 117)$ is equal to $(113^2 + 115^2 + 117^2 - 113 \times 115 - 115 \times 117 - 113 \times 117)$

(a) 0 (b) 4
(c) 8 (d) 12

(SSC CGL Mains Exam 2016)

482. When a number x is divided by a divisor it is seen that the divisor = 4 times the quotient = double the remainder. If the remainder is 80 then the value of x is

(a) 6480 (b) 9680
(c) 8460 (d) 4680

(SSC CGL Mains Exam 2016)

483. If P = 99, then the value of $P(P^2 + 3P + 3)$ is

(a) 9999 (b) 999999
(c) 99999 (d) 999999

(SSC CGL Mains Exam 2016)

484. If $x + \frac{1}{x} = c + \frac{1}{c}$ then the value of x

(a) c, 1/c (b) c, c^2
(c) c, 2c (d) 0,1

(SSC CGL Mains Exam 2016)

485. If the sum of square of two real numbers is 41 and their sum is 9. Then the sum of of cubes of these two numbers is

(a) 169 (b) 209
(c) 189 (d) 198

(SSC CGL Mains Exam 2016)

486. A complete factorisation of $x^4 + 64$ is

(a) $(x^2 + 8)^2$
(b) $(x^2 + 8)(x^2 - 8)$
(c) $(x^2 - 4x + 8)(x^2 - 4x - 8)$
(d) $(x^2 + 4x + 8)(x^2 - 4x + 8)$

(SSC CGL Mains Exam 2016)

487. If a + b = 1, then $a^4 + b^4 - a^3 - b^3 - 2a^2b^2 + ab$ is equal to

(a) 1 (b) 2
(c) 4 (d) 0

(SSC CGL Mains Exam 2016)

488. If $x^2 + y^2 + 6x + 5 = 4(x - y)$ then $x - y$ is

(a) 1 (b) –1
(c) 0 (d) 4

(SSC CGL Mains Exam 2016)

489. If a=299, b =298, c=297 then the value of $2a^3 + 2b^3 + 2c^3 - 6abc$ is

(a) 5154 (b) 5267
(c) 5364 (d) 5456

(SSC CGL Mains Exam 2016)

490. If $x + \frac{1}{x} = \sqrt{3}$ the value of $x^{15} + x^{12} + x^6 + 1$ is

(a) 0 (b) 1
(c) 2 (d) 3

(SSC CGL Mains Exam 2016)

491. If $x = 1 + \sqrt{2} + \sqrt{3}$, then the value of $2x^4 - 8x^3 - 5x^2 + 26x - 28$ is

(a) $2\sqrt{2}$ (b) $3\sqrt{3}$
(c) $5\sqrt{5}$ (d) $6\sqrt{6}$

(SSC CGL Mains Exam 2016)

492. If $2r = h + \sqrt{r^2 + h^2}$ then the ratio r : h $(r \neq 0)$ is

(a) 1 : 2 (b) 2 : 3
(c) 4 : 3 (d) 3 : 5

(SSC CGL Mains Exam 2016)

493. The simplified value of

$$\left(1 - \frac{2xy}{x^2 + y^2}\right) \div \left(\frac{x^3 - y^3}{x - y} - 3xy\right) is$$

(a) $\frac{1}{x^2 - y^2}$ (b) $\frac{1}{x^2 + y^2}$

(c) $\frac{1}{x - y}$ (d) $\frac{1}{x + y}$

(SSC CGL Mains Exam 2016)

494. If a+b+c = 0 then the value of

$$\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)} is$$

(a) 0 (b) 3 (c) 1 (d) 2

(SSC CGL Mains Exam 2016)

495. If $x^2 + y^2 + 2x + 1 = 0$, then the value of $x^{31} + y^{35}$ is

(a) -1 (b) 1 (c) 0 (d) 2

(SSC CGL Mains Exam 2016)

496. If $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$ the value of $\frac{x^2 + xy + y^2}{x^2 - xy + y^2}$ is

(a) 3/4 (b) 5/3
(c) 4/3 (d) 3/5

(SSC CGL Mains Exam 2016)

497. If $\left(x - \frac{1}{x}\right)^2 = 3$, then the value of $x^6 + \frac{1}{x^6}$ equals

(a) 90 (b) 110
(c) 100 (d) 120

(SSC CGL Mains Exam 2016)

498. If $x^4 + 2x^3 + ax^2 + bx + 9$ is a perfect square where a and b are positive real numbers, then the value of a and b

(a) a = 5, b = 6 (b) a = 6, b = 7
(c) a = 7, b = 7 (d) a = 7, b = 8

(SSC CGL Mains Exam 2016)

499. If $a^2+b^2+c^2 = 16$, $x^2+y^2+z^2 = 25$ and ax + by + cz = 20, then the value of $\frac{a+b+c}{x+y+z}$

(a) 3/5 (b) 4/5
(c) 5/3 (d) 5/4

(SSC CGL Mains Exam 2016)

500. The value of x which satisfies the equation

$$\frac{x + a^2 + 2c^2}{b+c} + \frac{x + b^2 + 2a^2}{c+a} + \frac{x + c + 2b^2}{a+b} = 0 \text{ is}$$

(a) $(a^2 + b^2 + c^2)$ (b) $(a^2 + 2b^2 + c^2)$
(c) $-(a^2 + b^2 + c^2)$ (d) $-(a^2 + b^2 + 2c^2)$

(SSC CGL Mains Exam 2016)

501. If $a^3 = 117 + b^3$ and a = 3 + b, then the value of a + b is :

(a) 7 (b) 13 (c) 49 (d) 0

(SSC CGL Mains Exam 2016)

502. If $a + \frac{1}{a} = -2$ then the value of $a^{1000} + a^{-1000}$ is

(a) 2 (b) 1 (c) 0 (d) 1\2

(SSC CGL Mains Exam 2016)

503. If $x^2 + \frac{1}{x^2} = 98$ (x > 0), then the value of $x^3 + \frac{1}{x^3}$ is

(a) 970 (b) 1030
(c) –970 (d) –1030

(SSC CGL Mains Exam 2016)

504. If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then the value of $c + \frac{1}{a}$ is

(a) 0 (b) 2 (c) 1 (d) 3

(SSC CGL Mains Exam 2016)

505. If $x = y + 2$ then $x^3 - y^3 - z^3$ is

(a) 0 (b) 3xyz

(c) –3xyz (d) 1

(SSC CGL Mains Exam 2016)

506. If a + b + c + d = 4 then the value of

$$\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)} + \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-d)(1-a)(1-b)}$$

is

(a) 0 (b) 1

(c) 4 (d) 1 + abcd

(SSC CGL Mains Exam 2016)

507. If x = 11, then value of $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$ is

(a) 11 (b) 10 (c) 12 (d) –10

(SSC CGL Mains Exam 2016)

508. If $a = \frac{1}{a-5}$ (a > 0) then the value of $a + \frac{1}{a}$ is

(a) $\sqrt{29}$ (b) $2\sqrt{29}$

(c) $-\sqrt{29}$ (d) $\sqrt{27}$

(SSC CGL Mains Exam 20

509. If $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$ (where $a \neq b \neq c$), then abc is equal to

(a) +1 (b) –1

(c) +1 & –1

(d) None of these

(SSC CGL Mains Exam 2016)

510. If ax + by = 1 and $bx + ay = \frac{2ab}{a^2+b^2}$ then $(x^2 + y^2)(a^2 + b^2)$ is equal to

(a) 1 (b) 2 (c) 0.5 (d) 0

(SSC CGL Mains Exam 2016)

511. If x, y, z are the three factors of $^3 - 7a - 6$, then value of x + y + ill be

(b) 3 (c) 6 (d) a

(SSC CGL Mains Exam 2016)

ANSWER KEY

1. (a)	52. (b)	103. (c)	154. (d)	205. (c)	256. (a)	308. (c)	360. (d)	412. (b)	464. (c)
2. (c)	53. (d)	104. (d)	155. (a)	206. (d)	257. (b)	309. (a)	361. (c)	413. (b)	465. (d)
3. (b)	54. (b)	105. (d)	156. (d)	207. (c)	258. (d)	310. (b)	362. (d)	414. (a)	466. (a)
4. (d)	55. (c)	106. (a)	157. (b)	208. (d)	259. (a)	311. (b)	363. (c)	415. (c)	467. (d)
5. (d)	56. (c)	107. (a)	158. (a)	209. (a)	260. (c)	312. (b)	364. (b)	416. (c)	468. (a)
6. (a)	57. (c)	108. (b)	159. (b)	210. (b)	261. (d)	313. (d)	365. (c)	417. (d)	469. (b)
7. (c)	58. (c)	109. (c)	160. (b)	211. (d)	262. (d)	314. (b)	366. (b)	418. (c)	470. (a)
8. (c)	59. (c)	110. (a)	161. (b)	212. (d)	263. (b)	315. (a)	367. (a)	419. (b)	471. (c)
9. (b)	60. (b)	111. (b)	162. (b)	213. (b)	264. (b)	316. (d)	368. (a)	420. (a)	472. (a)
10. (b)	61. (b)	112. (c)	163. (a)	214. (b)	265. (a)	317. (b)	369. (c)	421. (b)	473. (c)
11. (b)	62. (c)	113. (b)	164. (b)	215. (c)	266. (b)	318. (c)	370. (a)	422. (c)	474. (d)
12. (c)	63. (b)	114. (b)	165. (c)	216. (d)	267. (b)	319. (b)	371. (d)	423. (b)	475. (a)
13. (c)	64. (a)	115. (a)	166. (b)	217. (b)	268. (d)	320. (d)	372. (a)	424. (c)	476. (b)
14. (d)	65. (c)	116. (d)	167. (a)	218. (d)	269. (b)	321. (d)	373. (d)	425. (d)	477. (c)
15. (c)	66. (c)	117. (d)	168. (d)	219. (c)	270. (c)	322. (a)	374. (d)	426. (b)	478. (b)
16. (c)	67. (d)	118. (c)	169. (b)	220. (d)	271. (a)	323. (c)	375. (a)	427. (c)	479. (c)
17. (d)	68. (b)	119. (c)	170. (a)	221. (b)	272. (b)	324. (d)	376. (c)	428. (b)	480. (c)
18. (b)	69. (d)	120. (a)	171. (d)	222. (b)	273. (b)	325. (b)	377. (a)	429. (b)	481. (d)
19. (c)	70. (b)	121. (d)	172. (a)	223. (c)	274. (a)	326. (b)	378. (a)	430. (c)	482. (a)
20. (c)	71. (d)	122. (b)	173. (b)	224. (b)	275. (c)	327. (a)	379. (a)	431. (c)	483. (b)
21. (c)	72. (b)	123. (c)	174. (a)	225. (b)	276. (c)	328. (d)	380. (a)	432. (d)	484. (a)
22. (b)	73. (d)	124. (c)	175. (b)	226. (d)	277. (b)	329. (a)	381. (b)	433. (d)	485. (c)
23. (b)	74. (a)	125. (a)	176. (c)	227. (d)	278. (c)	330. (b)	382. (c)	434. (c)	486. (d)
24. (c)	75. (c)	126. (a)	177. (d)	228. (a)	279. (b)	331. (d)	383. (c)	435. (c)	487. (d)
25. (c)	76. (b)	127. (c)	178. (d)	229. (c)	280. (d)	332. (d)	384. (a)	436. (a)	488. (a)
26. (a)	77. (c)	128. (b)	179. (d)	230. (b)	281. (a)	333. (d)	385. (c)	437. (b)	489. (c)
27. (b)	78. (c)	129. (a)	180. (a)	231. (a)	282. (a)	334. (d)	386. (b)	438. (c)	490. (a)
28. (a)	79. (c)	130. (d)	181. (c)	232. (b)	283. (a)	335. (d)	387. (b)	439. (c)	491. (a)
29. (a)	80. (c)	131. (b)	182. (a)	233. (d)	284. (d)	336. (b)	388. (a)	440. (a)	492. (c)
30. (a)	81. (b)	132. (a)	183. (d)	234. (c)	285. (b)	337. (a)	389. (c)	441. (d)	493. (b)
31. (d)	82. (a)	133. (c)	184. (b)	235. (a)	286. (c)	338. (d)	390. (c)	442. (d)	494. (a)
32. (b)	83. (c)	134. (c)	185. (b)	236. (b)	287. (d)	339. (d)	391. (d)	443. (b)	495. (a)
33. (c)	84. (a)	135. (d)	186. (b)	237. (b)	288. (c)	340. (b)	392. (c)	444. (c)	496. (c)
34. (c)	85. (c)	136. (c)	187. (a)	238. (c)	289. (d)	341. (b)	393. (d)	445. (b)	497. (b)
35. (a)	86. (a)	137. (d)	188. (c)	239. (a)	290. (c)	342. (a)	394. (d)	446. (a)	498. (b)
36. (b)	87. (a)	138. (c)	189. (c)	240. (d)	291. (c)	343. (b)	395. (c)	447. (b)	499. (b)
37. (c)	88. (a)	139. (c)	190. (c)	241. (d)	292. (b)	344. (d)	396. (b)	448. (c)	450. (c)
38. (a)	89. (b)	140. (c)	191. (c)	242. (b)	293. (a)	345. (a)	397. (c)	449. (d)	501. (a)
39. (a)	90. (c)	141. (a)	192. (d)	243. (b)	294. (a)	346. (b)	398. (c)	450. (d)	502. (a)
40. (d)	91. (a)	142. (b)	193. (b)	244. (a)	295. (a)	347. (a)	399. (b)	451. (c)	503. (a)
41. (b)	92. (d)	143. (d)	194. (b)	245. (d)	296. (c)	348. (d)	400. (c)	452. (b)	504. (c)
42. (c)	93. (b)	144. (b)	195. (d)	246. (b)	297. (b)	349. (a)	401. (a)	453. (c)	505. (b)
43. (d)	94. (b)	145. (c)	196. (d)	247. (c)	298. (c)	350. (d)	402. (c)	454. (c)	506. (a)
44. (b)	95. (b)	146. (d)	197. (c)	248. (c)	299. (d)	351. (b)	403. (a)	455. (b)	507. (b)
45. (c)	96. (b)	147. (b)	198. (b)	249. (b)	300. (b)	352. (d)	404. (b)	456. (d)	508. (a)
46. (b)	97. (d)	148. (c)	199. (c)	250. (c)	301. (c)	353. (d)	405. (a)	457. (b)	509. (c)
47. (c)	98. (d)	149. (b)	200. (a)	251. (a)	302. (d)	354. (c)	406. (b)	458. (d)	510. (a)
48. (d)	99. (c)	150. (b)	201. (c)	252. (b)	303. (b)	355. (a)	407. (c)	459. (a)	511. (a)
49. (b)	100. (a)	151. (d)	202. (d)	253. (a)	304. (c)	356. (b)	408. (c)	460. (a)	
50. (d)	101. (b)	152. (d)	203. (b)	254. (b)	305. (b)	357. (b)	409. (a)	461. (d)	
51. (d)	102. (a)	153. (a)	204. (a)	255. (a)	306. (b)	358. (c)	410. (c)	462. (b)	
					307. (d)	359. (a)	411. (d)	463. (b)	

EXPLANATION

1. (a) $3 * 5 + 5 * 3$

$\Rightarrow 3 * 5 = 2 \times 3 - 3 \times 5 + 3 \times 5$
$= 6 - 15 + 15 = 6$

$\Rightarrow 5 * 3 = 2 \times 5 - 3 \times 3 + 3 \times 5$
$= 10 - 9 + 15 = 16$

$\therefore 3 * 5 + 5 * 3$

$\Rightarrow 6 + 16 = 22$

2. (c) $8 \times 2 = 8 + 2 + \frac{8}{2}$

$\Rightarrow 10 + 4 = 14$

3. (b) $x > y$

given :-

$x + y = 3(x - y)$

$\Rightarrow x + y = 3x - 3y$

$\Rightarrow x - 3x = -3y - y$

$\Rightarrow -2x = -4y \Rightarrow x = 2y$

$$\therefore \frac{3xy}{2(x^2 - y^2)} = \frac{3 \times 2y \times y}{2((2y)^2 - y^2)}$$

$$= \frac{6y^2}{2 \times (4y^2 - y^2)} = \frac{6y^2}{6y^2} = 1$$

4. (d)

$$\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x+1}\right)\left(1 + \frac{1}{x+2}\right)\left(1 + \frac{1}{x+3}\right)$$

Taking L.C.M of each term.

$$\Rightarrow \left(\frac{x+1}{x}\right)\left(\frac{x+1+1}{x+1}\right)\left(\frac{x+2+1}{x+2}\right)\left(\frac{x+3+1}{x+3}\right)$$

$$\Rightarrow \frac{1}{x} \times (x+4) \Rightarrow \frac{x+4}{x}$$

5. (d) $x = 7 - 4\sqrt{3}$

$$\frac{1}{x} \Rightarrow \frac{1}{7 - 4\sqrt{3}}$$

By rationalisation

$$\frac{1}{x} = \frac{1}{7 - 4\sqrt{3}} \times \frac{7 + 4\sqrt{3}}{7 + 4\sqrt{3}}$$

$$= \frac{7 + 4\sqrt{3}}{49 - 48} = 7 + 4\sqrt{3}$$

$$\therefore x + \frac{1}{x} = 7 - 4\sqrt{3} + 7 + 4\sqrt{3}$$

$= 14$

6. (a) $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$ (given)

or $\frac{c}{b} = \frac{5}{4}$

$$\therefore \frac{a+b}{b+c} = \frac{b\left(\frac{a}{b} + 1\right)}{b\left(\frac{c}{b} + 1\right)} = \frac{\frac{a}{b} + 1}{\frac{c}{b} + 1}$$

$$= \frac{\left(\frac{2}{3} + 1\right)}{\left(\frac{5}{4} + 1\right)} = \frac{\frac{5}{3}}{\frac{5+4}{4}}$$

$$= \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$$

$$\therefore \frac{a+b}{b+c} = \frac{20}{27}$$

Alternate:-

a	b	c
$2_{\times(4)}$	$3_{\times(4)}$	
	$4_{\times(3)}$	$5_{\times(3)}$
8	12	15

$$\frac{a+b}{b+c} = \frac{8+12}{12+15} = \frac{20}{27}$$

7. (c) $a * b = 2(a + b)$

$5 * 2 = 2(5 + 2) = 14$

8. (c) $\frac{2a+b}{a+4b} = 3$ (given)

$2a + b = 3(a+4b)$

$2a + b = 3a + 12b$

$\Rightarrow -a = 11b$

$a = -11b$

$$\therefore \frac{a+b}{a+2b}$$

$$\Rightarrow \frac{-11b+b}{-11b+2b}$$

$$= \frac{-10b}{-9b} = \frac{10}{9}$$

9. (b) $a * b = a + b + ab$

$3 * 4 = 3 + 4 + 3 \times 4 = 19$

$2 * 3 = 2 + 3 + 2 \times 3 = 5 + 6 = 11$

$\therefore 3 * 4 - 2 * 3 = 19 - 11 = 8$

10. (b) $x \otimes y = 3x + 2y$

$(2 \otimes 3) = 3 \times 2 + 2 \times 3 = 6 + 6 = 12$

$(3 \otimes 4) = 3 \times 3 + 2 \times 4 = 9 + 8 = 17$

$\therefore (2 \otimes 3) + (3 \otimes 4) = 12 + 17 = 29$

11. (b) $a : b = 2 : 3$, $b : c = 4 : 5$ (given)

a	:	b	:	c
$2_{\times(4)}$		$3_{\times(4)}$		
		$4_{\times(3)}$		$5_{\times(3)}$
8	:	12	:	15

$\therefore a^2 : b^2 : bc$

$\Rightarrow (8)^2 : (12)^2 : (12 \times 15)$

$64 : 144 : 180$

$\Rightarrow 16 : 36 : 45$

12. (c) $A : B = \frac{1}{2} : \frac{3}{8}$

$\Rightarrow 8 : 6$

$\Rightarrow 4 : 3$

$\Rightarrow B : C \Rightarrow \frac{1}{3} : \frac{5}{9}$

$\Rightarrow 9 : 15 \Rightarrow 3 : 5$

$\Rightarrow C : D \Rightarrow \frac{5}{6} : \frac{3}{4}$

$\Rightarrow 20 : 18 \Rightarrow 10 : 9$

A	:	B	:	C	:	D
4	:	3				
		3	:	5		
				10	:	9
8	:	6	:	10	:	9

13. (c) $A : B : C$

$2 : 3 : 4$

$$\therefore \frac{A}{B} : \frac{B}{C} : \frac{C}{A}$$

(Multiply with ABC)

$$\therefore \frac{A \times ABC}{B} : \frac{B \times ABC}{C} : \frac{C \times ABC}{A}$$

$A^2C : B^2A : BC^2$

$\Rightarrow (2)^2 \times 4 : (3)^2 \times 2 : 3 \times (4)^2$

$\Rightarrow 16 : 18 : 48$

$\Rightarrow 8 : 9 : 24$

Alternate:-

$$\frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : \frac{4}{2}$$

$$\Rightarrow \frac{2}{3} \times 12 : \frac{3}{4} \times 12 : \frac{4}{2} \times 12$$

$\Rightarrow 8 : 9 : 24$

14. (d) A : B B : C C : D
1 : 2 3 : 4 5 : 6

A	B	C	
$1_{\times 3}$	$2_{\times 3}$		
	$3_{\times 2}$	$4_{\times 2}$	
3	6	8	
A	B	C	D
$3_{\times 5}$	$6_{\times 5}$	$8_{\times 5}$	
		$5_{\times 8}$	$6_{\times 8}$
15	30	40	48

$\therefore$ D : C : B : A
48 : 40 : 30 : 15

15. (c) $\frac{2a-5b}{3a+6b} = \frac{4}{7}$

$\Rightarrow 7(2a - 5b) = 4(3a + 6b)$

$\Rightarrow 14a - 35b = 12a + 24b$

$\Rightarrow 2a = 59b$

$\frac{a}{b} = \frac{59}{2}$

$\therefore a : b = 59 : 2$

16. (c) $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$

$\therefore a = 3k$
$b = 4k$
$c = 7k$

$\therefore \frac{a+b+c}{c}$

$= \frac{3k+4k+7k}{7k} = 2$

17. (d) $\frac{144}{0.144} = \frac{14.4}{x}$

$\Rightarrow \frac{144 \times 1000}{144} = \frac{144}{x \times 10}$

$\Rightarrow 1000 = \frac{144}{10x}$

$\Rightarrow x = \frac{144}{1000 \times 10}$

$x = \frac{144}{10000} = 0.0144$

18. (b) $1 < x < 2$

$\sqrt{(x-1)^2} + \sqrt{(3-x)^2}$

(square root cancel with square)
$\therefore x - 1 + 3 - x = 2$

19. (c) $a \oplus b = (a \times b) + b$

$5 \oplus 7 = (5 \times 7) + 7 = 35 + 7 = 42$

20. (c) $10^{0.48} = x$
$10^{0.70} = y$
and $x^z = y^2$

$\therefore \left(10^{0.48}\right)^z = \left(10^{0.70}\right)^2$

$\Rightarrow 10^{0.48z} = 10^{1.40}$ (If $a^x = a^y$, if base equal power are equal: $(x = y)$
$\therefore 0.48z = 1.40$

$z = \frac{140}{48} = \frac{35}{12} = 2.9$

21. (c) $47.2506 = 4A + 7B + 2C + \frac{5}{D} + 6E$

$47.2506 = 4\times10 + 7\times1 + 2\times 0.1000 + 5\times0.0100 + 0 + 6\times0.0001$

$\therefore A = 10 \quad B = 1 \quad C = 0.1000$

$D = \frac{1}{\frac{1}{100}} = 100, \quad E = 0.0001$

$\therefore 5A + 3B + 6C + D + 3E$
$= 5 \times 10 + 3 \times 1 + 6 \times 0.1 + 100 + 3 \times 0.0001$
$= 50 + 3 + 0.6 + 100 + 0.0003$
$= 153.6003$

22. (b) $x * y = x^2 + y^2 - xy$
$9*11 = (9)^2 + (11)^2 - 11 \times 9$
$81 + 121 - 99 = 103$

23. (b) $5^{5x+5} = 1$
$5^{5x+5} = 5^0 \ (a^0 = 1)$
[If bases are same then power are equal]
$5x + 5 = 0$
$5x = -5$
$x = -1$

24. (c) $3^{x+3} + 7 = 250$
$3^{x+3} = 250 - 7$
$3^{x+3} = 243$
$3^{x+3} = 3^5$
$x + 3 = 5$
$x = 2$

25. (c) $\frac{1}{4} \times \frac{2}{6} \times \frac{3}{8} \times \frac{4}{10} \times \frac{5}{12} \times \ldots\ldots \times$

$\frac{31}{64} = \frac{1}{2^x}$

$\Rightarrow \left(\frac{1}{2}\right)^{30} \times \left(\frac{1}{2}\right)^6$

$= \left(\frac{1}{2}\right)^{30+6} = \frac{1}{2^x}$

or $\frac{1}{2^{36}} = \frac{1}{2^x}$

$\therefore x = 36$

26. (a) $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow \therefore x = \frac{1}{y}$

$x = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{\left(\sqrt{3}+1\right)^2}{3-1}$

$= \frac{3+1+2\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{2}$

$= \left(2+\sqrt{3}\right)$

$x^2 = \left(2+\sqrt{3}\right)^2 = 4 + 3 + 4\sqrt{3}$

$= 7 + 4\sqrt{3}$

$y^2 = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$y^2 = \frac{7-4\sqrt{3}}{49-48} = \frac{7-4\sqrt{3}}{1} = 7 - 4\sqrt{3}$

$\therefore x^2 + y^2 = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$
$= 14$

Alternate:-

$x^2 + y^2 = x^2 + \frac{1}{x^2}$

$= \left(x + \frac{1}{x}\right)^2 - 2 \quad \left(\therefore x = \frac{1}{y}\right)$

$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 - 2$

$= \left[\frac{\left(\sqrt{3}+1\right)^2 + \left(\sqrt{3}-1\right)^2}{3-1}\right]^2 - 2$

$\therefore (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$= \left[\frac{2\left(\left(\sqrt{3}\right)^2 + (1)^2\right)}{2}\right]^2 - 2$

$= (3 + 1)^2 - 2 = 16 - 2 = 14$

27. (b) $x = 3 + 2\sqrt{2}$

$x = 2 + 1 + 2\sqrt{2} = \left(\sqrt{2}+1\right)^2$

$\sqrt{x} = \sqrt{2} + 1$

$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1}$

$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{\sqrt{2}-1}{1} = \sqrt{2} - 1$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}}$

$= \sqrt{2} + 1 - \left(\sqrt{2} - 1\right)$

$= \sqrt{2} + 1 - \sqrt{2} + 1 = 2$

28. (a) P = 999

$\sqrt[3]{p\left(p^2+3p+3\right)+1}$

$\Rightarrow \sqrt[3]{p^3+3p^2+3p+1}$

$\therefore \sqrt[3]{(p+1)^3}$

$= \sqrt[3]{(999+1)^3}$

$= \sqrt[3]{(1000)^3} = 1000$

29. (a) $\frac{a}{b} = \frac{7}{9}$, $\frac{b}{c} = \frac{3}{5}$

$\therefore$ a : b : c

7 : 9

$3_{\times 3}$: $5_{\times 3}$

7 : 9 : 15

30. (a) x : y

7 : 3

$\therefore \frac{xy+y^2}{x^2-y^2} = \frac{21+9}{49-9} = \frac{30}{40} = \frac{3}{4}$

31. (d) Given [p] means greatest positive integer less than or [p] equal to p,

$\Rightarrow$ [p] = p

$\Rightarrow$ [−p] = p

$\Rightarrow \left[-\frac{1}{4}\right] + \left[4-\frac{1}{4}\right] + [3]$

$= \frac{1}{4} + 4 - \frac{1}{4} + 3 = 7$

32. (b) $\frac{(243)^{\frac{n}{5}}.3^{2n+1}}{9^n.3^{n-1}} = \frac{\left(3^5\right)^{\frac{n}{5}}.3^{2n+1}}{3^{2n}.3^{n-1}}$

$= \frac{3^{n+2n+1}}{3^{2n+n-1}} = \frac{3^{3n+1}}{3^{3n-1}} = 3^{3n+1-3n+1}$

$= 3^2 = 9$

33. (c) $x = 0.5$

y = 0.2

$\sqrt{0.6} \times (3y)^x = \sqrt{0.6} \times (3\times 0.2)^{0.5}$

$= \sqrt{0.6} \times \sqrt{0.6} = 0.6$

34. (c) $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x$

$x^{x\sqrt{x}} = \left(x^{\frac{3}{2}}\right)^x$

$x^{x\sqrt{x}} = x^{\frac{3}{2}x}$

(If bases are same then their power is also same)

$\therefore x\sqrt{x} = \frac{3}{2}x$ or $\sqrt{x} = \frac{3}{2}$

$x = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

35. (a) $a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$= \frac{1}{2}\left[(7-5)^2+(5-3)^2+(3-7)^2\right]$

$= \frac{1}{2}(4+4+16) = \frac{24}{2} = 12$

36. (b) $7^x = \frac{1}{343} = \frac{1}{7^3}$

$7^x = \frac{1}{7^3}$

$7^x = 7^{-3}$

$x = -3$ (If bases are equal then their power are also equal)

37. (c)

$$\frac{\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}-3\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{5}-\left(\frac{1}{3}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{3}\right)}$$

$A^3 + B^3 + C^3 - 3ABC = (A + B + C)(A^2 + B^2 + C^2 - AB - BC - CA)$

$$\therefore \frac{\left(\frac{1}{3}\right)^3+\left(\frac{1}{4}\right)^3-3\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}+\left(\frac{1}{5}\right)^3}{\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{5}\right)^2-\frac{1}{3}\times\frac{1}{4}-\frac{1}{4}\times\frac{1}{5}-\frac{1}{5}\times\frac{1}{3}}$$

$$= \frac{\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)\left[\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{5}\right)^2-\frac{1}{3}\times\frac{1}{4}-\frac{1}{4}\times\frac{1}{5}-\frac{1}{5}\times\frac{1}{3}\right]}{\left[\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{5}\right)^2-\frac{1}{3}\times\frac{1}{4}-\frac{1}{4}\times\frac{1}{5}-\frac{1}{5}\times\frac{1}{3}\right]}$$

$= \frac{20+15+12}{60} = \frac{47}{60}$

38. (a) $0.13 \times p^2 = 13$

$p^2 = \frac{13}{0.13} = \frac{13}{13}\times 100$

p = 10

39. (a) $\frac{x}{y} = \frac{3}{4}$, $\frac{6}{7}+\frac{y-x}{y+x} = ?$

$= \frac{6}{7} + \frac{y\left(1-\frac{x}{y}\right)}{y\left(1+\frac{x}{y}\right)}$

$= \frac{6}{7} + \frac{\left(1-\frac{3}{4}\right)}{\left(1+\frac{3}{4}\right)}$

$= \frac{6}{7}+\left[\frac{4-3}{4}\times\frac{4}{(4+3)}\right]$

$= \frac{6}{7}+\frac{1}{7}=\frac{7}{7} = 1$

Alternate:-

$\frac{6}{7}+\frac{y-x}{y+x} = \frac{6}{7} + \frac{4-3}{4+3}$

$= \frac{6}{7} + \frac{1}{7} = \frac{7}{7} = 1$

40. (d) $x = 7 - 4\sqrt{3}$

$= 4 + 3 - 4\sqrt{3}$

$= (2)^2 + \left(\sqrt{3}\right)^2 - 2 \times 2\sqrt{3}$

$= \left(2-\sqrt{3}\right)^2$

$\therefore$ $[(a^2 + b^2 - 2ab = (a-b)^2]$

$\Rightarrow x = \left(2-\sqrt{3}\right)^2$

$\sqrt{x} = 2-\sqrt{3}$

$\frac{1}{\sqrt{x}} = \frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= 2+\sqrt{3}$

$\therefore \sqrt{x}+\frac{1}{\sqrt{x}}$

$= 2 - \sqrt{3} + 2 + \sqrt{3} = \mathbf{4}$

41. (b) $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$

$\therefore a = \frac{1}{b}$

$a + b = a + \frac{1}{a}$

$\Rightarrow \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$

$\Rightarrow \frac{5+1+2\sqrt{5}+5+1-2\sqrt{5}}{\left(\sqrt{5}\right)^2-(1)^2}$

$\Rightarrow \frac{6+2\sqrt{5}+6-2\sqrt{5}}{5-1} = \frac{12}{4} = 3$

$\therefore \frac{a^2+ab+b^2}{a^2-ab+b^2}$

$= \frac{a^2+\frac{1}{a^2}+ab}{a^2+\frac{1}{a^2}-ab}$

$\Rightarrow a + \frac{1}{a} = 3$

$a^2 + \frac{1}{a^2} = 9 - 2 = 7$ (ab = 1)

$\therefore \frac{a^2+\frac{1}{a^2}+ab}{a^2+\frac{1}{a^2}-ab} = \frac{7+1}{7-1} = \frac{8}{6} = \frac{4}{3}$

42. (c) a = 4.36

b = 2.39

c = 1.97

a − b − c

= 4.36 − 2.39 − 1.97

= 0

$a^3 - b^3 - c^3 - 3abc$

$= \frac{1}{2}(a-b-c)[(a-b)^2+(b-c)^2+(c-a)^2]$

= 0

43. (d) $\frac{3a+5b}{3a-5b} = 5$

$\Rightarrow$ 3a + 5b = 15 a – 25b

$\Rightarrow$ 12a = 30b

$\Rightarrow$ 2a = 5b

a : b

5 : 2

44. (b) p : q = r : s = t : u = 2 : 3

$\therefore \frac{mp+nr+ot}{mq+ns+ou}$

$\Rightarrow \frac{m\times 2x + n\times 2x + o\times 2x}{m\times 3x + n\times 3x + o\times 3x}$

$\Rightarrow \frac{2x(m+n+o)}{3x(m+n+o)} = \frac{2}{3}$

$\therefore$ mp + nr + ot : mq + ns + ou

$\boxed{2:3}$

45. (c) x : y = 3 : 4

$\frac{7x+3y}{7x-3y} = \frac{y}{y}\left(\frac{7\frac{x}{y}+3}{7\frac{x}{y}-3}\right) = \frac{7\times\frac{3}{4}+3}{7\times\frac{3}{4}-3}$

$= \frac{\frac{21}{4}+3}{\frac{21}{4}-3} = \frac{\frac{21+12}{4}}{\frac{21-12}{4}} = \frac{11}{3}$

46. (b) $x + \frac{1}{4}\sqrt{x} + a^2$

$= \left(\sqrt{x}\right)^2 + 2\times\frac{1}{8}\times\sqrt{x} + a^2$

[$(A^2 + 2BA + B^2) = (A + B)^2$]

Here, A = $\sqrt{x}$ and B = a

B = $\frac{1}{8}$ $\therefore$ a = $\frac{1}{8}$

47. (c) Given that a $\neq$ b Let a = 16, b = 4

$\therefore$ by options

So, $\frac{a+b}{2} = \frac{16+4}{2} = 10$

and $\sqrt{ab} = \sqrt{16\times 4} = 8$

$\therefore \frac{a+b}{2} > \sqrt{ab}$

$\therefore$ option (c) is correct.

48. (d) $x^{1/3} = y^{1/4}$

$\Rightarrow$ LCM of 3, 4 = 12

$\therefore \left(x^{1/3}\right)^{12} \Rightarrow \left(y^{1/4}\right)^{12}$

$x^4 = y^3$

take power '5' on both sides

$\Rightarrow \left(x^4\right)^5 = \left(y^3\right)^5$

$\Rightarrow x^{20} = y^{15}$

49. (b) $x = \frac{\sqrt{3}}{2}$

or $1 + x = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$

$= \frac{2(2+\sqrt{3})}{2\times 2}$

(divides and multiply by 2)

$\Rightarrow 1 + x = \frac{4+2\sqrt{3}}{4}$

$= \frac{1+3+2\sqrt{3}}{4}$

$= \frac{(1)^2 + \left(\sqrt{3}\right)^2 + 2\times 1\times\sqrt{3}}{4}$

$1 + x \Rightarrow \frac{\left(1+\sqrt{3}\right)^2}{4}$

$\therefore \sqrt{1+x} = \frac{1+\sqrt{3}}{2}$

Similarly,

$\sqrt{1-x} \Rightarrow \frac{\sqrt{3}-1}{2}$

$\therefore \frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}}$

$= \frac{\frac{1+\sqrt{3}}{2}}{1+\frac{1+\sqrt{3}}{2}} + \frac{\frac{\sqrt{3}-1}{2}}{1-\frac{\sqrt{3}-1}{2}}$

$= \frac{1+\sqrt{3}}{3+\sqrt{3}} + \frac{\sqrt{3}-1}{3-\sqrt{3}}$

$= \frac{1+\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+1\right)} + \frac{\sqrt{3}-1}{\sqrt{3}\left(\sqrt{3}-1\right)}$

$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

50. (d) $x^2 - 4x - 1 = 0$

$x^2 - 1 = 4x$ (divide x both sides)

$x - \frac{1}{x} = 4$

$x^2 + \frac{1}{x^2} - 2 = 16$

$x^2 + \frac{1}{x^2} = 18$

51. (d) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x^2+\frac{1}{x^2}+1\right)$

$= \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$

$\because \boxed{\begin{array}{l}(A+B)\left(A^2-AB+B^2\right) = A^3+B^3\\ (A-B)\left(A^2+AB+B^2\right) = A^3-B^3\end{array}}$

$= \left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right) = \boxed{x^6-\frac{1}{x^6}}$

52. (b) $a^{2x+2} = 1$

$a^{2x+2} = a^0$

$2x + 2 = 0$

$x = -\frac{2}{2} = -1$

53. (d) $x^2 - x - 1$

$\Rightarrow$ if $ax^2 + bx + c = 0$

Then minimum value of equation is $\left(\frac{4ac-b^2}{4a}\right)$

$\because$ in the given equation,

a = 1

b = –1

c = –1

$\therefore$ minimum value

$\Rightarrow \frac{(4\times 1\times(-1)) - (-1)^2}{4\times 1}$

$= \frac{-4-1}{4} = -\frac{5}{4}$

54. (b) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

L.H.S = $\frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2}$

(Rationalisation)

$= \frac{\left(\sqrt{7}-2\right)^2}{\left(\sqrt{7}\right)^2-(4)} = \frac{7+4-4\sqrt{7}}{7-4}$

$= \frac{11-4\sqrt{7}}{3}$

$\frac{11}{3} - \frac{4}{3}\sqrt{7} = -\frac{4}{3}\sqrt{7} + \frac{11}{3}$

= $a\sqrt{7}$+b = R.H.S

Compare the cofficients of $\sqrt{7}$ and constant term

a = $-\frac{4}{3}$

b = $\frac{11}{3}$

55. (c) $a+\frac{1}{b}=1$, $b+\frac{1}{c}=1$, $c+\frac{1}{a}$ = ?

Put values,

a = $\frac{1}{2}$ b = 2 c = –1

$c+\frac{1}{a} = -1 + \frac{1}{\left(\frac{1}{2}\right)} = -1 + 2 = 1$

Alternate:

$\Rightarrow a + \frac{1}{b} = 1$ .....(i)

$\Rightarrow a = 1 - \frac{1}{b} = \boxed{\frac{b-1}{b}}$

$\frac{1}{a} = \frac{b}{b-1} \Rightarrow b + \frac{1}{c} = 1$

$\frac{1}{c} = 1 - b, \boxed{c = \frac{1}{1-b}}$

$\therefore c + \frac{1}{a} = \frac{1}{1-b} + \frac{b}{b-1}$

$= \frac{1}{1-b} - \frac{b}{1-b} = \frac{1-b}{1-b} = 1$

56. (c) $x = \sqrt{3} + \sqrt{2}$

$\therefore \frac{1}{x} = \sqrt{3} - \sqrt{2}$

$x^3 + \frac{1}{x^3}$

$\therefore x^3 = \left(\sqrt{3} + \sqrt{2}\right)^3$

$= \left(\sqrt{3}\right)^3 + \left(\sqrt{2}\right)^3 + 3 \times \sqrt{3} \times \sqrt{2}\left(\sqrt{3} + \sqrt{2}\right)$

$= 3\sqrt{3} + 2\sqrt{2} + 3\sqrt{6}\left(\sqrt{3} + \sqrt{2}\right)$

$= 3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} + 6\sqrt{3}$

$x^3 = 9\sqrt{3} + 11\sqrt{2}$

$\frac{1}{x^3} = 9\sqrt{3} - 11\sqrt{2}$

$x^3 + \frac{1}{x^3} = 9\sqrt{3} + 11\sqrt{2} + 9\sqrt{3} - 11\sqrt{2}$

$= 18\sqrt{3}$

Alternate:-

$x = \sqrt{3} + \sqrt{2}$

$\frac{1}{x} = \sqrt{3} - \sqrt{2}$

and $x + \frac{1}{x} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}$

$= 2\sqrt{3}$

$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$= \left(2\sqrt{3}\right)^3 - 3\left(2\sqrt{3}\right)$

$= 24\sqrt{3} - 6\sqrt{3} = 18\sqrt{3}$

57. (c) $x + y = 7$ (cubing both sides)

$(x + y)^3 = (7)^3$

$x^3 + y^3 + 3(x + y)xy = 343$

$x^3 + y^3 + 21xy = 343$

58. (c) given $a \otimes b = 2a$

where $a > b$

$a \otimes b = a + b$

where $a < b$

$a \otimes b = a^2$

where $a = b$

$= \frac{(5+7)+(4)^2}{3(5)^2 - (2\times15) - 3}$

$= \frac{12+16}{75-30-3} = \frac{28}{42} = \frac{2}{3}$

59. (c) $(125)^x = 3125$

$(5^3)^x = 5^5$

$5^{3x} = 5^5$

$\therefore 3x = 5$

$x = \mathbf{\frac{5}{3}}$

60. (b) $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$

By option put $x = 4$

$\Rightarrow 5^{\sqrt{4}} + 12^{\sqrt{4}} = 13^{\sqrt{4}}$

$\Rightarrow 5^2 + 12^2 = 13^2$

$\Rightarrow 169 = 169$

hence, $x = 4$

61. (b) $n + \frac{2}{3}n + \frac{1}{2}n + \frac{1}{7}n = 97$

$\Rightarrow \frac{42n + 28n + 21n + 6n}{42} = 97$

$\Rightarrow \frac{97n}{42} = 97$

$\Rightarrow n = 42$

62. (c) $x = 3 + \sqrt{8}$

$\frac{1}{x} = \frac{1}{3 + \sqrt{8}}$

$\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$

$= \frac{3 - \sqrt{8}}{9 - 8} = 3 - \sqrt{8}$

$x + \frac{1}{x} = 3 + \sqrt{8} + 3 - \sqrt{8} = 6$

$x + \frac{1}{x} = 6$

squaring both sides

$x^2 + \frac{1}{x^2} + 2 = 36$

$x^2 + \frac{1}{x^2} = 34$

63. (b) $x - \frac{1}{x} = 4$

$x^2 + \frac{1}{x^2} - 2 = 16$

(On Squaring)

$\Rightarrow x^2 + \frac{1}{x^2} = 18$

$x^2 + \frac{1}{x^2} + 2 - 2 = 18$

$x^2 + \frac{1}{x^2} + 2 = 20$

$\left(x + \frac{1}{x}\right)^2 = 20$

$x + \frac{1}{x} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

64. (a) $4b^2 + \frac{1}{b^2} = 2$

$(2b)^2 + \left(\frac{1}{b}\right)^2 + 4 - 4 = 2$

$\left(2b + \frac{1}{b}\right)^2 - 4 = 2$

$\left(2b + \frac{1}{b}\right)^2 = 6$

$2b + \frac{1}{b} = \sqrt{6}$

Take cube both sides

$\left(2b + \frac{1}{b}\right)^3 = (\sqrt{6})^3$

$8b^3 + \frac{1}{b^3} + 3 \times 2b \times \frac{1}{b}\left(2b + \frac{1}{b}\right) = 6\sqrt{6}$

$8b^3 + \frac{1}{b^3} + 6\sqrt{6} = 6\sqrt{6}$

$8b^3 + \frac{1}{b^3} = 0$

65. (c) $2^{2x-y} = 16$ (given)

$\Rightarrow 2^{2x-y} = 2^4$

$\therefore 2x - y = 4$ (i)

$2^{x+y} = 32$ (given)

$2^{x+y} = 2^5$

and $x + y = 5$ (ii)

by equation (i) and (ii)

$2x - y = 4$

$x + y = 5$

$3x = 9$

$x = 3$

$\therefore y = 2$ $xy = 3 \times 2 = 6$

66. (c) $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$

$\Rightarrow \left(\frac{3}{5}\right)^{+3-6} = \left(\frac{3}{5}\right)^{2x-1}$

$\Rightarrow -3 = 2x - 1$

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$

67. (d) $a^2 - b^2 = 19$

$(a - b)(a + b) = 19 \times 1$

$a + b = 19 \longrightarrow$ (I)

$a - b = 1 \longrightarrow$ (II)

$2a = 20$

$\boxed{a = 10}$

(Adding equation (i) and (ii))

68. (b) $\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}} = \frac{2}{1}$

(by c–d rule)

$\Rightarrow \frac{\sqrt{3+x}}{\sqrt{3-x}} = \frac{2+1}{2-1} = \frac{3}{1}$

$\left(\frac{A}{B}=\frac{C}{D}, \frac{A+B}{A-B}=\frac{C+D}{C-D}\right)$

$\frac{\sqrt{3+x}}{\sqrt{3-x}} = 3$

Squaring both sides

$\frac{3+x}{3-x} = 9$

$3 + x = 27 - 9x$

$10x = 24$

$x = \frac{24}{10} = \frac{12}{5}$

69. (d) $x = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} \times \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$= \frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)^2}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$

$= \frac{1+x+1-x+2\sqrt{1-x^2}}{1+x-1+x}$

$= \frac{2+2\sqrt{1-x^2}}{2x} = \frac{1+\sqrt{1-x^2}}{x}$

$= \frac{1+\sqrt{1-\frac{3}{4}}}{\sqrt{3}} \times 2$

$= \frac{\left(1+\frac{1}{2}\right)}{\sqrt{3}} \times 2 = \frac{\frac{3}{2}\times 2}{\sqrt{3}} = \sqrt{3}$

70. (b) $4^{4x+1} = \frac{1}{64}$

$= 4^{4x+1} = \frac{1}{(4)^3}$

$\Rightarrow 4^{4x+1} = (4)^{-3}$

$\Rightarrow 4x + 1 = -3$

$\Rightarrow 4x = -4$

$\Rightarrow x = -1$

71. (d) $\frac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}-\sqrt{x-4}} = \frac{2}{1}$

by C – D rule

$\Rightarrow \frac{\sqrt{x+4}}{\sqrt{x-4}} = \frac{2+1}{2-1} = \frac{3}{1}$

$\Rightarrow \left(\frac{\sqrt{x+4}}{\sqrt{x-4}}\right)^2 = \left(\frac{3}{1}\right)^2$

$\Rightarrow \frac{x+4}{x-4} = 9$again C & D rule

$\Rightarrow \frac{x}{4} = \frac{9+1}{9-1}$

$\Rightarrow \frac{x}{4} = \frac{10}{8}$

$\Rightarrow x = \frac{10}{8} \times 4 = 5$

72. (b) $\sqrt{2^x} = 256$

$\sqrt{2^x} = 2^8$

$2^x = (2^8)^2$

$= 2^x = 2^{16}$

$\boxed{x = 16}$

73. (d) $\left(\sqrt{5}\right)^7 \div \left(\sqrt{5}\right)^5 = 5^P$

$\Rightarrow \frac{\left(\sqrt{5}\right)^7}{\left(\sqrt{5}\right)^5} = 5^P$

$= \left(\sqrt{5}\right)^2 = 5^P = 5^1 = 5^P$

$\boxed{P = 1}$

74. (a) 1.5a = 0.04b

$\frac{a}{b} = \frac{0.04}{1.5} = \frac{4}{100} \times \frac{10}{15} = \frac{2}{75}$

Let a = 2x, b = 75x

$\therefore \frac{b-a}{b+a} = \frac{75x-2x}{75x+2x} = \frac{73}{77}$

Alternate:-

$\frac{a}{b} = \frac{0.04}{1.5}$

$\frac{b-a}{b+a} = \frac{1.5-0.04}{1.5+0.04} = \frac{1.46}{1.54} = \frac{73}{77}$

75. (c) $x = \left(\sqrt{2}+1\right)^{1/3}$

Take cube on both sides

$\Rightarrow x^3 = \sqrt{2} + 1$

$\Rightarrow \frac{1}{x^3} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{\sqrt{2}-1}{1}$

$\frac{1}{x^3} = \sqrt{2} - 1$

$\Rightarrow x^3 - \frac{1}{x^3} = \sqrt{2} + 1 - \sqrt{2} + 1 = 2$

76. (b) $\frac{x^2 - x + 1}{x^2 + x + 1} = \frac{3}{2}$ Given

$\Rightarrow \frac{x\left\{\left(x+\frac{1}{x}\right)-1\right\}}{x\left\{\left(x+\frac{1}{x}\right)+1\right\}} = \frac{3}{2}$

Let $\left(x + \frac{1}{x} = y\right)$

$\Rightarrow \frac{\left(x+\frac{1}{x}\right)-1}{\left(x+\frac{1}{x}\right)+1} = \frac{3}{2}$

$\frac{y-1}{y+1} = \frac{3}{2}$

$\Rightarrow 2(y - 1) = 3(y + 1)$

$2y - 2 = 3y + 3$

$y = -2 - 3 = -5$

$\therefore x + \frac{1}{x} = -5$

77. (c) $x = 3 + \sqrt{8}$

$x^2 = 9 + 8 + 2 \times 3\sqrt{8}$

$x^2 = 17 + 6\sqrt{8}$

$\frac{1}{x^2} = 17 - 6\sqrt{8}$

$x^2 + \frac{1}{x^2} = 17 + 6\sqrt{8} + 17 - 6\sqrt{8} = 34$

78. (c) $x = 5 + 2\sqrt{6}$

$x = 3 + 2 + 2\sqrt{3} \times \sqrt{2}$

$x = \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 + 2\sqrt{3} \times \sqrt{2}$

$x = \left(\sqrt{3}+\sqrt{2}\right)^2$

$\sqrt{x} = \sqrt{3} + \sqrt{2}$

Similarly

$\Rightarrow \frac{1}{\sqrt{x}} = \sqrt{3} - \sqrt{2}$

$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}$

$= 2\sqrt{3}$

79. (c) a > b

a + b = 5

a × b = 6 Assume a = 3

$a^2 - b^2$ Values b = 2

= 9 – 4 = 5

Alternate:-

$a + b = 5$ given

Squaring both sides

$\Rightarrow (a + b)^2 = (5)^2$

$a^2 + b^2 + 2ab = 25$

$ab = 6$ given

$= a^2 + b^2 + 2 \times 6 = 25$

$= a^2 + b^2 = 13$

$\Rightarrow a^2 + b^2 + 2ab - 2ab = 13$

$a^2 + b^2 - 2ab = 13 - 2ab$

$(a - b)^2 = 13 - 2 \times 6$

$a - b = 1$

$\therefore (a + b)(a - b) = 5 \times 1$

$a^2 - b^2 = 5$

80. (c) $x + \frac{9}{x} = 6$

Take values of x

Let $x = 3$

$3 + \frac{9}{3} = 6$ (Proved)

So, $x = 3$

$\therefore x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 10$

Alternate:-

$x + \frac{9}{x} = 6$

On squaring

$\left(x + \frac{9}{x}\right)^2 = 36$

$x^2 + \frac{81}{x^2} + 2 \times \frac{9}{x} \times x = 36$

$x^2 + \frac{81}{x^2} - 18 = 0$

$\left(x - \frac{9}{x}\right)^2 = 0$

$x = \frac{9}{x}$

$x^2 = 9$

Hence $x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 10$

81. (b) $2p + \frac{1}{p} = 4$

Divide by 2

$\frac{2p}{2} + \frac{1}{2p} = \frac{4}{2}$

$p + \frac{1}{2p} = 2$

Take cube on both sides

$\Rightarrow \left(p + \frac{1}{2p}\right)^3 = (2)^3$

$p^3 + \frac{1}{8p^3} + 3 \times p \times \frac{1}{2p}\left(p + \frac{1}{2p}\right) = 8$

$p^3 + \frac{1}{8p^3} + \frac{3}{2} \times 2 = 8$

$p^3 + \frac{1}{8p^3} = 8 - 3 = 5$

82. (a) $a^6 + b^6 = (a^2)^3 + (b^2)^3$

$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$

$\therefore a^6 + b^6 = (a^2 + b^2) \times 0 = 0$

83. (c) $x + \frac{1}{x} = 3$ (Given)

$\frac{x^3 + \frac{1}{x}}{x^2 - x + 1}$ (divide by x)

$\Rightarrow \frac{\frac{x^3}{x} + \frac{1}{x^2}}{\frac{x^2}{x} - \frac{x}{x} + \frac{1}{x}} = \frac{x^2 + \frac{1}{x^2}}{x - 1 + \frac{1}{x}}$

$= \frac{x^2 + \frac{1}{x^2}}{x + \frac{1}{x} - 1}$

$\therefore x + \frac{1}{x} = 3$

$\therefore x^2 + \frac{1}{x^2} = 9 - 2 = 7$

$\therefore \frac{x^2 + \frac{1}{x^2}}{x + \frac{1}{x} - 1} = \frac{7}{3 - 1} = \frac{7}{2}$

84. (a) $a + \frac{1}{a} + 1 = 0$

$a + \frac{1}{a} = -1$

Squaring both sides

$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 1$

$\Rightarrow a^2 + \frac{1}{a^2} = -1$

$\Rightarrow a^2 + 1 = \frac{-1}{a^2}$ ....(i)

$\Rightarrow a + \frac{1}{a} = -1$ (Given)

$\therefore a^2 + 1 = -a$ .....(ii)

$\Rightarrow -a = \frac{-1}{a^2}$

From equation (i) and (ii)

$a^3 = 1$

$\therefore a^3 - 1 = 0$

$\Rightarrow a^4 - a = a(a^3 - 1) = a \times 0 = 0.$

85. (c) $x = a + \frac{1}{a}$

$y = a - \frac{1}{a}$

$\therefore (x + y) = a + \frac{1}{a} + a - \frac{1}{a} = 2a$

$\therefore (x - y) = a + \frac{1}{a} - a + \frac{1}{a} = \frac{2}{a}$

$\therefore x^4 + y^4 - 2x^2y^2 = (x^2 - y^2)^2$

$\Rightarrow ((x + y)(x - y))^2$

$\Rightarrow \left(2a \times \frac{2}{a}\right)^2$

$= (4)^2 = 16$

86. (a) $a = 11$

$b = 9$

$\Rightarrow \frac{a^2 + b^2 + ab}{a^3 - b^3}$

$\Rightarrow (a^3 - b^3) = ((a - b)(a^2 + ab + b^2))$

$\therefore \frac{a^2 + b^2 + ab}{(a - b)(a^2 + ab + b^2)}$

$= \frac{1}{a - b} = \frac{1}{11 - 9} = \frac{1}{2}$

87. (a) $p = 101$

$= \sqrt[3]{p(p^2 - 3p + 3) - 1}$

$= \sqrt[3]{p^3 - 3p^2 + 3p - 1}$

$\therefore [(p - 1)^3 = p^3 - (1)^3 - 3p(p - 1)]$

$= \sqrt[3]{(p-1)^3}$

$= p - 1 = 101 - 1 = 100$

88. (a) $x = 19$ $y = 18$

$\frac{x^2 + y^2 + xy}{x^3 - y^3}$

$= \frac{x^2 + y^2 + xy}{(x - y)(x^2 + y^2 + xy)}$

$= \frac{1}{x - y} = \frac{1}{19 - 18} = 1$

89. (b) $50\%(p - q) = 30\%(p + q)$

$\frac{p - q}{2} = \frac{3}{10}(p + q)$

$\left[50\% = \frac{1}{2}\right]$

$\Rightarrow 5(p - q) = 3(p + q)$

$\Rightarrow 5p - 5q = 3p + 3q$

$\Rightarrow 2p = 8q$

$\Rightarrow 1p = 4q$

$\Rightarrow p : q$

$\Rightarrow 4 : 1$

90. (c) $x : y$
$2 : 1$
then, $5x^2 - 13xy + 6y^2$
$\Rightarrow 5 \times 4 - 13 \times 2 \times 1 + 6 \times 1^2$
$\Rightarrow 20 - 26 + 6 = 0$

91. (a) $\frac{a}{3} = \frac{b}{2} \Rightarrow \frac{a}{b} = \frac{3}{2}$

$$\frac{2a + 3b}{3a - 2b} = \frac{2 \times 3 + 3 \times 2}{3 \times 3 - 2 \times 2}$$

$$= \frac{6 + 6}{9 - 4} = \frac{12}{5}$$

92. (d) $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$

Add 3 both sides

$$\Rightarrow \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + 3 = 1 + 3$$

$$\Rightarrow \left(\frac{a}{1-a} + 1\right) + \left(\frac{b}{1-b} + 1\right) + \left(\frac{c}{1-c} + 1\right) = 4$$

$$\Rightarrow \left(\frac{a + 1 - a}{1 - a}\right) + \left(\frac{b + 1 - b}{1 - b}\right) + \left(\frac{c + 1 - c}{1 - c}\right) = 4$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

93. (b)

$\frac{2x-y}{x+2y} \times \frac{1}{2}$ (Cross Multiply)

$\Rightarrow 4x - 2y = x + 2y$
$3x = 4y$
$x : y = 4 : 3$

$$\Rightarrow \frac{3x - y}{3x + y} = \frac{3 \times 4 - 3}{3 \times 4 + 3}$$

$$= \frac{12 - 3}{12 + 3} = \frac{9}{15} = \frac{3}{5}$$

94. (b) $x + \frac{1}{x} = 5$

$$\therefore \frac{2x}{3x^2 - 5x + 3} \text{ (Divide by } x\text{)}$$

$$= \frac{\frac{2x}{x}}{\frac{3x^2}{x} - \frac{5x}{x} + \frac{3}{x}} = \frac{2}{3x + \frac{3}{x} - 5}$$

$$= \frac{2}{3\left(x + \frac{1}{x}\right) - 5} = \frac{2}{3 \times 5 - 5}$$

$$= \frac{2}{10} = \frac{1}{5}$$

95. (b) $\sqrt{1 - \frac{x^3}{100}} = \frac{3}{5}$

$$\Rightarrow 1 - \frac{x^3}{100} = \left(\frac{3}{5}\right)^2$$

$$\Rightarrow 1 - \frac{9}{25} = \frac{x^3}{100}$$

$$\Rightarrow \frac{16}{25} = \frac{x^3}{100}$$

$$\Rightarrow \frac{16 \times 100}{25} = x^3$$

$\Rightarrow 16 \times 4 = x^3$
$\Rightarrow 4 = x$

96. (b) $\sqrt{1 + \frac{x}{9}} = \frac{13}{3}$

By option
Put $x = 160$

$$\sqrt{1 + \frac{160}{9}} = \sqrt{\frac{169}{9}} = \frac{13}{3}$$

Alternate:
Squaring both sides

$$\left(\sqrt{1 + \frac{x}{9}}\right)^2 = \left(\frac{13}{3}\right)^2$$

$$\Rightarrow 1 + \frac{x}{9} = \frac{169}{9}$$

$$\Rightarrow \frac{9 + x}{9} = \frac{169}{9}$$

$9 + x = 169$

$\boxed{x = 160}$

97. (d) $\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} = a + b\sqrt{6}$

$$\Rightarrow \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{16 \times 3} + \sqrt{9 \times 2}}$$

$$= \frac{4\sqrt{3} + 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}}$$

$$\Rightarrow \frac{4\sqrt{3} + 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}} \times \frac{4\sqrt{3} - 3\sqrt{2}}{4\sqrt{3} - 3\sqrt{2}}$$

$$= \frac{\left(4\sqrt{3} + 5\sqrt{2}\right)\left(4\sqrt{3} - 3\sqrt{2}\right)}{48 - 18}$$

$$= \frac{8\sqrt{6} + 18}{30} = \frac{8\sqrt{6}}{30} + \frac{18}{30}$$

$$= \frac{4}{15}\sqrt{6} + \frac{3}{5}$$

$$= \frac{3}{5} + \frac{4}{15}\sqrt{6}$$

$$\therefore \frac{3}{5} + \frac{4}{15}\sqrt{6} = a + b\sqrt{6}$$

By comparing cofficients of rational and irrational parts.

$$\Rightarrow a = \frac{3}{5} \quad b = \frac{4}{15}$$

$$\left(\frac{3}{5}, \frac{4}{15}\right)$$

98. (d) $a * b = ab$
$5 * 3 = 5 \times 3 = 15$

99. (c) $\sqrt{0.03 \times 0.3a} = 0.3 \times 0.3\sqrt{b}$

Squaring both sides
$0.03 \times 0.3\, a = (0.3)^2 \times (0.3)^2\, b$

$$\frac{3}{100} \times \frac{3}{10} a = \frac{9}{100} \times \frac{9}{100} \times b$$

$$9a = \frac{81}{10} b \Rightarrow 10a = 9b$$

$$\frac{a}{b} = \frac{9}{10} = 0.9$$

100. (a) $x * y = (x + 3)^2 (y - 1)$
$5 * 4 = (5 + 3)^2 (4 - 1)$
$= 64 \times 3 = 192$

101. (b) $9\sqrt{x} = \sqrt{12} + \sqrt{147}$

$9\sqrt{x} = \sqrt{4 \times 3} + \sqrt{49 \times 3}$

$9\sqrt{x} = 9\sqrt{3}$

$x = 3$

102. (a) $\sqrt{1 + \frac{x}{961}} = \frac{32}{31}$

(Squaring both sides)

$$\Rightarrow 1 + \frac{x}{961} = \frac{1024}{961}$$

$$\Rightarrow \frac{961 + x}{961} = \frac{1024}{961}$$

$x = 1024 - 961 = 63$

103. (c) $\sqrt{0.04 \times 0.4 \times a} = 0.004 \times 0.4 \times \sqrt{b}$

$$\Rightarrow \sqrt{\frac{4}{100} \times \frac{4}{10} \times a} = \frac{4}{1000} \times \frac{4}{10} \sqrt{b}$$

$$\Rightarrow \frac{4}{10}\sqrt{\frac{a}{10}} = \frac{4}{10} \times \frac{4}{1000}\sqrt{b}$$

$$\frac{a}{10} = \frac{16}{1000000} b$$

$$\Rightarrow \frac{a}{b} = \frac{16 \times 10}{1000000} = \frac{16}{100000}$$

$$\frac{a}{b} = 16 \times 10^{-5}$$

104. (d) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{3}{1}$

$\Rightarrow \frac{2\times 9 + 3\times 9 + 4\times 9}{2\times 1 + 3\times 1 + 4\times 1}$

$= \frac{18 + 27 + 36}{2 + 3 + 4} = \frac{81}{9} = \mathbf{9}$

105. (d) $2x + \frac{1}{3x} = 5$

$\Rightarrow 6x^2 + 1 = 15x$

$\therefore \frac{5x}{6x^2 + 20x + 1}$

$= \frac{5x}{15x + 20x} = \frac{5x}{35x} = \frac{1}{7}$

106. (a) $x \propto \frac{1}{y^2 - 1}$ (Given)

$x = k \times \frac{1}{y^2 - 1}$

(k is constant)

Now $x = 24$ when $y = 10$ given

$\Rightarrow 24 = k \times \frac{1}{(10)^2 - 1}$

$\Rightarrow 24 = \frac{k}{99}$

$k = 24 \times 99$

$\Rightarrow x = ?$

$y = 5$

$x = 24\times 99\times \frac{1}{25-1}$

$= 24\times 99\times \frac{1}{24}$ $x = 99$

107. (a) $x^2 + y^2 + 2x + 1 = 0$

$\Rightarrow x^2 + 2x + 1 + y^2 = 0$

$(x + 1)^2 + y^2 = 0$

Hence both terms are squares and there addition is zero so, it can be possible only when both terms are zeros.

$\therefore x + 1 = 0$

$\Rightarrow x = -1$

$y = 0$

$\therefore x^{31} + y^{35}$

$= (-1)^{31} + (0)^{35} = -1$

108. (b) $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$

$\frac{\frac{x}{x}}{2\frac{x^2}{x} + \frac{5x}{x} + \frac{2}{x}} = \frac{1}{6}$

$\Rightarrow \frac{1}{2x + \frac{2}{x} + 5} = \frac{1}{6}$

$\Rightarrow 2\left(x + \frac{1}{x}\right) + 5 = 6$

$\Rightarrow 6 - 5 = 2\left(x + \frac{1}{x}\right)$

$\Rightarrow 1 = 2\left(x + \frac{1}{x}\right)$

$\Rightarrow x + \frac{1}{x} = \frac{1}{2}$

109. (c) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$\Rightarrow a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$

$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$

$\Rightarrow (a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$a = 1$

$b = -1$

$c = -1$

$\therefore 2a - 3b + 4c$

$= 2 \times 1 - 3 \times (-1) + 4\times (-1)$

$= 2 + 3 - 4 = 1$

110. (a) $(3a + 1)^2 + (b - 1)^2 + (2c - 3)^2 = 0$

$\Rightarrow (3a + 1)^2 = 0$

$\Rightarrow 3a = -1 \Rightarrow a = \frac{-1}{3}$

$(b - 1)^2 = 0 \Rightarrow b - 1 = 0$

$\Rightarrow b = 1$

$(2c - 3) = 0 \Rightarrow c = \frac{3}{2}$

$\therefore 3a + b + 2c = 3\times \frac{-1}{3} + 1 + \frac{3}{2}\times 2$

$= -1 + 1 + 3 = \mathbf{3}$

111. (b) $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$

Now

$\Rightarrow \frac{(a-b)^2}{(b-c)(c-a)} \times \frac{(a-b)}{a-b}$

Multiply divide by (a – b) in Ist term

$\Rightarrow$ (b – c) in IInd term

$\Rightarrow$ (c – a) in IIIrd term

$\Rightarrow \frac{(a-b)^2(a-b)}{(b-c)(c-a)(a-b)} +$

$\frac{(b-c)^2(b-c)}{(a-b)(b-c)(c-a)} +$

$\frac{(c-a)^2(c-a)}{(a-b)(b-c)(c-a)}$

$\Rightarrow$ Let $a - b = x$

$b - c = y$

$c - a = z$

$\therefore x + y + z = 0$

$\therefore x^3 + y^3 + z^3 = 3xyz$

$\therefore (a - b)^3 + (b - c)^3 + (c - a)^3$

$= 3(a - b)(b - c)(c - a)$

$\therefore \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

112. (c) $(a - 3)^2 + (b - 4)^2 + (c - 9)^2 = 0$

$a - 3 = 0$ $a = 3$

$b - 4 = 0$ $b = 4$

$c - 9 = 0$ $c = 9$

$\therefore \sqrt{a + b + c} = \sqrt{3 + 4 + 9}$

$= \sqrt{16} = \pm\mathbf{4}$

113. (b) $1.5x = 0.04y$

$\Rightarrow \frac{x}{y} = \frac{0.04}{1.5} = \frac{4}{100}\times\frac{10}{15} = \frac{2}{75}$

$\therefore \frac{y^2 - x^2}{y^2 + x^2 + 2xy} = \frac{(y-x)(y+x)}{(y+x)^2}$

$\Rightarrow \frac{y - x}{y + x} = \frac{75 - 2}{75 + 2} = \frac{73}{77}$

114. (b) $a^{1/3} = 11$, $a = 11^3 = 1331$

$a^2 - 331a = a(a - 331)$

$= 1331(1331 - 331)$

$= 1331\times 1000 = 1331000$

115. (a) $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 4$

Take $x = y = 1$

$1 + 1 + \frac{1}{1} + \frac{1}{1} = 4$

Hence

$x^2 + y^2 = 1 + 1 = 2$

116. (d) $a^2 + b^2 = 2$

$c^2 + d^2 = 1$

Put values of a, b, c, d

Take $a = b = 1$

$c = 1$

$d = 0$

$\Rightarrow (ad - bc)^2 + (ac + bd)^2$

$\Rightarrow (0-1)^2 + (1 + 0)^2$

$\Rightarrow (-1)^2 + (1)^2 = \mathbf{2}$

117. (d) $x = \frac{4ab}{a + b}$

$\Rightarrow \frac{x}{2a} = \frac{2b}{a + b}$

$\frac{x + 2a}{x - 2a} = \frac{2b + a + b}{2b - a - b} = \frac{3b+a}{b-a}$

(By C – D rule)

$\Rightarrow$ again

$\frac{x}{2b} = \frac{2a}{a + b}$

$\frac{x + 2b}{x - 2b} = \frac{2a + a + b}{2a - a - b} = \frac{3a + b}{a - b}$

$\Rightarrow \frac{x + 2a}{x - 2a} + \frac{x + 2b}{x - 2b} = \frac{3b + a}{b - a} + \frac{3a + b}{a - b}$

$\Rightarrow \dfrac{3b + a - 3a - b}{b - a}$

$\Rightarrow \dfrac{2b - 2a}{b - a} = \dfrac{2(b-a)}{(b-a)} = 2$

118. (c) $m + \dfrac{1}{m-2} = 4$

$\Rightarrow (m - 2) + \dfrac{1}{(m-2)} = 2$

$\Rightarrow$ Squaring both sides

$(m - 2)^2 + \dfrac{1}{(m-2)^2} + 2 \times (m - 2)$

$\times \dfrac{1}{(m-2)} = 4$

$(m - 2)^2 + \dfrac{1}{(m-2)^2} = 2$

119. (c) $a^2 + b^2 + 2b + 4a + 5 = 0$

$a^2 + b^2 + 2b + 4a + 4 + 1 = 0$

$a^2 + 4a + 4 + b^2 + 2b + 1 = 0$

$(a + 2)^2 + (b + 1)^2 = 0$

$a + 2 = 0 \qquad a = -2$

$b + 1 = 0 \qquad b = -1$

$\dfrac{a-b}{a+b} \Rightarrow \dfrac{-2+1}{-2-1}$

$\Rightarrow \dfrac{-1}{-3} = \dfrac{1}{3}$

120. (a) $x - y = \dfrac{x+y}{7} = \dfrac{xy}{4} = k$ (let)

$x - y = k$ .....(i)

$x + y = 7k$ .....(ii)

$xy = 4k$ ....(iii)

$\therefore x - y = k$ .....(i)

$x + y = 7k$ ...(ii)

$x = 4k$

$y = 3k$

$\therefore xy \Rightarrow 4k \times 3k = 12k^2$

$12k^2 = 4k$

$k = \dfrac{1}{3} \quad \therefore xy = 4k = \dfrac{4}{3}$

121. (d) $\dfrac{x^2}{yz} + \dfrac{y^2}{zx} + \dfrac{z^2}{xy}$

$= \dfrac{x^2}{yz} \times \dfrac{x}{x} + \dfrac{y^2 \times y}{zx \times y} + \dfrac{z^2}{xy} \times \dfrac{z}{z}$

$= \dfrac{x^3 + y^3 + z^3}{xyz}$

($\therefore$ *If* $x + y + z = 0$

then, $x^3 + y^3 + z^3 \Rightarrow 3xyz$)

$\therefore \dfrac{3xyz}{xyz} = 3$

122. (b) $a + b + c = 0$

$\dfrac{1}{(a+b)(b+c)} + \dfrac{1}{(a+c)(b+a)} + \dfrac{1}{(c+a)(c+b)}$

$\Rightarrow \dfrac{(a+c)+(b+c)+(a+b)}{(a+b)(a+c)(b+c)}$

$\Rightarrow \dfrac{2(a+b+c)}{(a+b)(a+c)(b+c)} = 0$

($\therefore a + b + c = 0$)

123. (c) $a + b + c = 0$

Assume values $a = 2, b = -2, c = 0$

$a + b + c = 2 - 2 + 0 = 0$(satisfy)

$\therefore \dfrac{a^2 + b^2 + c^2}{a^2 - bc}$

$\Rightarrow \dfrac{4+4+0}{4-0} \Rightarrow \dfrac{8}{4} = 2$

Alternate:

$a + b + c = 0$

$b + c = -a$

Squaring both sides

$(b + c)^2 = a^2$

$b^2 + c^2 + 2bc = a^2$

$b^2 + c^2 = a^2 - 2bc$

$\therefore \dfrac{a^2 + b^2 + c^2}{a^2 - bc}$

$\Rightarrow \dfrac{a^2 + a^2 - 2bc}{a^2 - bc}$

$\Rightarrow \dfrac{2a^2 - 2bc}{a^2 - bc} \Rightarrow \dfrac{2(a^2 - bc)}{a^2 - bc} = 2$

124. (c) $x^2 + y^2 - 4x - 4y + 8 = 0$

$x^2 + 4 - 4x + y^2 + 4 - 4y = 0$

$(x - 2)^2 + (y - 2)^2 = 0$

$x - 2 = 0, \qquad y - 2 = 0$

$x = 2, \qquad y = 2$

$\therefore x - y = 2 - 2 = 0$

125. (a) $x = b + c - 2a$

$y = c + a - 2b$

$z = a + b - 2c$

$\Rightarrow x + y + z = (b + c - 2a) + (c + a - 2b) + (a + b - 2c) = 0$

$\therefore$ Now

$= x^2 + y^2 + 2xy - z^2$

$= (x + y)^2 - z^2 \; (A^2 - B^2) = (A + B)(A - B)$

$(x + y - z)(x + y + z)$

= As we know $(x + y + z) = 0$

$\therefore x^2 + y^2 - z^2 + 2xy = 0 \times (x + y - z) = 0$

126. (a) $x + \dfrac{1}{x} = \sqrt{3}$ (take cube on both sides)

$\left(x + \dfrac{1}{x}\right)^3 = \left(\sqrt{3}\right)^3$

$x^3 + \dfrac{1}{x^3} + 3x \times \dfrac{1}{x}\left(x + \dfrac{1}{x}\right) = 3\sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3\left(\sqrt{3}\right) = 3\sqrt{3}$

$x^3 + \dfrac{1}{x^3} = 0$

$\therefore x^6 = -1$

$\therefore x^{18} + x^{12} + x^6 + 1$

$= (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

127. (c) $ax^3 + 3x^2 - 8x + b$ is divisible by $(x + 2)$ and $(x - 2)$

$\therefore (x + 2)$ and $(x - 2)$ are factors

$\therefore x + 2 = 0 \Rightarrow x = -2$

$x - 2 = 0 \Rightarrow x = 2$

Put $x = -2$

$\therefore a(-2)^3 + 3(-2)^2 - 8(-2) + b = 0$

$= -8a + 12 + 16 + b = 0$

$-8a + b + 28 = 0$

$-8a + b = -28$ ....(I)

and

Put $x = 2$

$\Rightarrow a(2)^3 + 3(2)^2 - 8 \times 2 + b = 0$

$\Rightarrow 8a + 12 - 16 + b = 0$

$8a + b - 4 = 0$

$8a + b = 4$ .....(II)

From equation (I) & (II)

$\therefore -8a + b = -28$

$8a + b = 4$

$2b = -24$

$b = -12$

$a = 2$

128. (b) $x^2 - 3x + 1 = 0$

$x^2 + 1 = 3x$

Divide by x

$= \dfrac{x^2}{x} + \dfrac{1}{x}$

$\Rightarrow \dfrac{3x}{x}$

$x + \dfrac{1}{x} = 3$

Cubing both sides

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3x \times \dfrac{1}{x}\left(x + \dfrac{1}{x}\right) = 27$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3 \times 3 = 27$

$x^3 + \dfrac{1}{x^3} = 18$

129. (a) $x + \dfrac{1}{4x} = \dfrac{3}{2}$

Multiply by 2 both sides

$\therefore 2x + \dfrac{1}{2x} = 3$

Take cube both sides

$= \left(2x + \dfrac{1}{2x}\right)^3 = (3)^3$

$= 8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}\left(2x + \frac{1}{2x}\right)$

$= 27$

$= 8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$

$8x^3 + \frac{1}{8x^3} = 27 - 9 = 18$

130. (d) $x^4 + \frac{1}{x^4} = 119 \qquad x > 1$

$\therefore x^4 + \frac{1}{x^4} + 2 = 119 + 2 = 121$

$\left(x^2 + \frac{1}{x^2}\right)^2 = (11)^2$

$x^2 + \frac{1}{x^2} = 11$

$x^2 + \frac{1}{x^2} + 2 = 11 + 2$

$\left(x + \frac{1}{x}\right)^2 = 13$

$x + \frac{1}{x} = \sqrt{13}$

$\Rightarrow$ Taking cube both sides

$x^3 + \frac{1}{x^3} + 3\sqrt{13} = \left(\sqrt{13}\right)^3$

$x^3 + \frac{1}{x^3} + 3\sqrt{13} = 13\sqrt{13}$

$x^3 + \frac{1}{x^3} = 10\sqrt{13}$

131. (b) $3x + \frac{1}{2x} = 5$

$\Rightarrow$ Multiply both sides by $\frac{2}{3}$

$\therefore 3x \times \frac{2}{3} + \frac{1}{2}x \times \frac{2}{3} = 5 \times \frac{2}{3}$

$2x + \frac{1}{3x} = \frac{10}{3}$

$\therefore$ Taking cube on both sides

$8x^3 + \frac{1}{27x^3} + 3 \times 2x \times \frac{1}{3x}\left(2x + \frac{1}{3x}\right)$

$= \left(\frac{10}{3}\right)^3$

$8x^3 + \frac{1}{27x^3} + 2 \times \frac{10}{3} = \frac{1000}{27}$

$8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$

$= \frac{1000 - 180}{27} = \frac{820}{27} = 30\frac{10}{27}$

132. (a) $x + y = z$

$x + y - z = 0$

If $a + b + c = 0$

then $a^3 + b^3 + c^3 - 3abc = 0$

$\therefore x^3 + y^3 - z^3 = -3xyz$

$\therefore x^3 + y^3 - z^3 + 3xyz = 0$

$3xyz - 3xyz = 0$

133. (c) $\frac{a}{b} + \frac{b}{a} = 1$

$\therefore a^2 + b^2 = ab$

$\therefore a^2 + b^2 - ab = 0$

$\therefore a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (a + b) \times 0 = 0$

134. (c) $x = 2 - 2^{1/3} + 2^{2/3}$

$x - 2 = 2^{2/3} - 2^{1/3} \qquad ....(I)$

Take cube both sides

$(x - 2)^3 = (2^{2/3} - 2^{1/3})^3$

$x^3 - 8 - 6x(x - 2) = (2^{2/3})^3 - (2^{1/3})^3$

$- 3 \times 2^{2/3}.2^{1/3}(2^{2/3} - 2^{1/3})$

$x^3 - 8 - 6x^2 + 12x$

$= 2^2 - 2 - 3 \times 2^{\frac{2+1}{3}}(x - 2)$

From equation (I)

$\therefore x^3 - 8 - 6x^2 + 12x$

$= 4 - 2 - 3 \times 2(x - 2)$

$x^3 - 8 - 6x^2 + 12x = 2 - 6x + 12$

$x^3 + 18x - 6x^2 - 8 - 14 = 0$

$x^3 + 18x - 6x^2 - 22 = 0$

$\therefore x^3 - 6x^2 + 18x + 18 = 22 + 18 = 40$

135. (d) $a^3 - b^3 - c^3 - 3abc = 0$

$\therefore a - b - c = 0$

$a = b + c$

136. (c) $a = 2.361$

$b = 3.263$

$c = 5.624$

$a + b - c = 0$

$2.361 + 3.263 - 5.624 = 0$

$\therefore a^3 + b^3 - c^3 + 3abc \Rightarrow 0$

137. (d) $p = 124$

$\sqrt[3]{p\left(p^2 + 3p + 3\right) + 1}$

$= \sqrt[3]{p^3 + 3p^2 + 3p + 1}$

$= \sqrt[3]{(p + 1)^3} = \sqrt[3]{(125)^3} = 125$

138. (c) $x + \frac{1}{x} = 2$

(assume $x = 1$, so, $1 + 1 = 2$)

$x^{17} + \frac{1}{x^{19}} = (1)^{17} + \frac{1}{(1)^{19}}$

$= 1 + 1 = 2$

139. (c) $x : y = 3 : 4$

$\therefore \frac{5x - 2y}{7x + 2y} = \frac{5 \times 3 - 2 \times 4}{7 \times 3 + 2 \times 4}$

$= \frac{15 - 8}{21 + 8} = \frac{7}{29}$

140. (c) $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$

$\frac{\frac{2p}{p}}{\frac{p^2}{p} - \frac{2p}{p} + \frac{1}{p}} = \frac{1}{4}$

$\frac{2}{p + \frac{1}{p} - 2} = \frac{1}{4}$

$p + \frac{1}{p} - 2 = 8$

$p + \frac{1}{p} = 10$

141. (a) $x + y = 2z$

$x - z = z - y$

$x - z \Rightarrow -(y - z) \qquad .....(i)$

$\frac{x}{x - z} + \frac{z}{y - z} = \frac{x}{x - z} - \frac{z}{x - z}$

$= \frac{x - z}{x - z} = 1$

142. (b) Let,

$a = 1$

$b = 180$

$c = 1$

$\therefore a^3b = abc = 180$ (satisfied)

$\therefore c = 1$

143. (d) $(a - 1)\sqrt{2} + 3 = b\sqrt{2} + a$

Comparing Cofficient of $\sqrt{2}$ & constant terms.

$\therefore a = 3$

$\therefore a - 1 = b$

$3 - 1 = b$

$b = 2$

$a + b = 3 + 2 = 5$

144. (b) $64^{x+1} = \frac{64}{4^x}$

$\left(4^3\right)^{x+1} = \frac{4^3}{4^x}$

$4^{3x+3} = 4^{3-x}$

$\therefore 3x + 3 = 3 - x$

$4x = 0$

$x = 0$

145. (c) $ax^2 + bx + c = a(x - p)^2$

$ax^2 + bx + c = a(x^2 + p^2 - 2px)$

$\Rightarrow ax^2 + bx + c = ax^2 + ap^2 - 2apx$

Comparing cofficients of x^2 and x

$\Rightarrow b = -2ap$

$\Rightarrow p = \frac{-b}{2a} \qquad ....(i)$

and $c = ap^2$

$\Rightarrow c = a \times \frac{b^2}{4a^2}$ (From (i))

$\boxed{4ac = b^2}$

146. (d) $a + b + c + d = 1$

$(1 + a)(1 + b)(1 + c)(1 + d)$

$\Rightarrow$ For maximum value a, b, c, d

$a = b = c = d = \frac{1}{4}$

$= \left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{4}\right) = \left(\frac{5}{4}\right)^4$

147. (b) $a^2 + b^2 + c^2 + 3 = 2(a + b + c)$
$a^2 + b^2 + c^2 + 3 = 2a + 2b + 2c$
$a^2 - 2a + 1 + b^2 - 2b + 1 + c^2 - 2c + 1 = 0$
$(a - 1)^2 + (b - 1)^2 + (c - 1)^2 = 0$
$a = 1$
$b = 1$
$c = 1$
$(a + b + c) \Rightarrow 1 + 1 + 1 = 3$

148. (c) $x - \frac{1}{x} = 5$

$x^2 + \frac{1}{x^2} - 2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = 27$

149. (b) $n = 7 + 4\sqrt{3}$
$n = 4 + 3 + 4\sqrt{3}$
$n = (2)^2 + \left(\sqrt{3}\right)^2 + 2 \times 2 \times \sqrt{3}$
$n = \left(2 + \sqrt{3}\right)^2$
$\sqrt{n} = 2 + \sqrt{3}$
$\therefore \frac{1}{\sqrt{n}} = 2 - \sqrt{3}$
$\sqrt{n} + \frac{1}{\sqrt{n}} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

150. (b) $x = \sqrt{3} + \sqrt{2}$
$\frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \sqrt{3} - \sqrt{2}$
$x + \frac{1}{x} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2} = 2\sqrt{3}$

151. (d) $p + q = 10$ .... (i)
and $pq = 5$
Squaring both sides of equation (i)
$(p + q)^2 = (10)^2$
$p^2 + q^2 + 2pq = 100$
$p^2 + q^2 + 2 \times 5 = 100$
$p^2 + q^2 = 90$
Now,
$\therefore \frac{p}{q} + \frac{q}{p} = \frac{p^2 + q^2}{pq} = \frac{90}{5} = 18$

152. (d) $x = 3 + 2\sqrt{2}$, $xy = 1$, $y^2 = \frac{1}{x^2}$
$y = \frac{1}{x} = \frac{1}{3 + 2\sqrt{2}} = 3 - 2\sqrt{2}$
$\therefore x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$
$\therefore x^2 + \frac{1}{x^2} = 36 - 2 = 34$
$= \frac{x^2 + 3xy + y^2}{x^2 - 3xy + y^2} = \frac{x^2 + \frac{1}{x^2} + 3}{x^2 + \frac{1}{x^2} - 3}$
$= \frac{34 + 3}{34 - 3} = \frac{37}{31}$

153. (a) $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$ (given)
$\therefore \frac{x}{b+c} = \frac{y}{c+a}$
$\Rightarrow \frac{x}{b+c} = \frac{x-y}{(b+c)-(c+a)} = \frac{x-y}{b-a}$
Similarly
$\frac{y}{c+a} = \frac{z}{a+b}$
$\therefore \frac{y}{c+a} = \frac{y-z}{(c+a)-(a+b)} = \frac{y-z}{c-b}$ again
$\Rightarrow \frac{z}{a+b} = \frac{x}{b+c}$
$\Rightarrow \frac{z}{a+b} = \frac{z-x}{(a+b)-(b+c)} = \frac{z-x}{a-c}$
$\therefore \frac{x-y}{b-a} = \frac{y-z}{c-b} = \frac{z-x}{a-c}$

154. (d) $x - y = 2$, $xy = 24$ (given)
by squaring
$x^2 + y^2 - 2xy = 4$
$x^2 + y^2 - 2 \times 24 = 4$
$x^2 + y^2 = 4 + 48 = 52$

155. (a) $\frac{x^2}{y^2} + tx + \frac{y^2}{4}$ (given)
To make it a perfect square it should be in the form
$A^2 \pm 2AB + B^2 = (A \pm B)^2$
$= \left(\frac{x}{y}\right)^2 \pm tx + \left(\frac{y}{2}\right)^2$
$= A^2 \pm 2AB + B^2$
$A = \frac{x}{y}$, $B = \frac{y}{2}$ & $2AB = tx$
So, $tx = \pm 2 \times \frac{x}{y} \times \frac{y}{2}$
$tx = \pm x$
$t = \pm 1$

156. (d) $a = x + y$
$b = x - y$
$c = x + 2y$
$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}((a - b)^2 + (b - c)^2 + (c - a)^2)$
$= \frac{1}{2}((x + y - x + y)^2 + (x - y - x - 2y)^2 + (x + 2y - x - y)^2)$
$= \frac{1}{2}((2y)^2 + (-3y)^2 + y^2)$
$= \frac{1}{2}(4y^2 + 9y^2 + y^2)$
$= \frac{1}{2}(14y^2) = 7y^2$

157. (b) $x + \frac{1}{x} = 2$, $x \neq 0$
put $x = 1$
$1 + 1 = 2$
$\therefore x^2 + \frac{1}{x^3} = 1 + 1 = 2$

158. (a) $\left(x + \frac{1}{x}\right)^2 = 3$
$x + \frac{1}{x} = \sqrt{3}$
$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$
$\Rightarrow x^3 + \frac{1}{x^3} = 0$
$\Rightarrow x^6 + 1 = 0$
$\Rightarrow x^6 = -1$
$\Rightarrow x^{72} + x^{66} + x^{54} + x^{24} + x^6 + 1$
$\Rightarrow \left(x^6\right)^{12} + \left(x^6\right)^{11} + \left(x^6\right)^9 + \left(x^6\right)^4 + x^6 + 1$
$\Rightarrow (-1)^{12} + (-1)^{11} + (-1)^9 + (-1)^4 + -1 + 1$
$\Rightarrow 1 - 1 - 1 + 1 - 1 + 1 = 0$

159. (b) $a + \frac{1}{a} = \sqrt{3}$
$a^6 = -1$
$\therefore a^6 - \frac{1}{a^6} + 2 = -1 - \frac{1}{(-1)} + 2$
$= -1 + 1 + 2 = 2$

160. (b) $x^3 + y^3 = 35$
$\Rightarrow x + y = 5$
Take cube on both sides,
$(x + y)^3 = (5)^3$
$x^3 + y^3 + 3xy(x + y) = 125$
$35 + 3xy(5) = 125$
$15xy = 125 - 35$
$15xy = 90$
$xy = 6$
$\therefore \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{5}{6}$

161. (b) $a^3 - b^3 = 56$
$\Rightarrow a - b = 2$
$\Rightarrow a^3 - b^3 - 3ab(a - b) = (2)^3$
(By cubing)
$56 - 3ab \times 2 = 8$
$-6ab = 8 - 56$
$6ab = 48$
$\Rightarrow ab = 8$
$(a - b) = 2$
$(a - b)^2 = a^2 + b^2 - 2ab = 4$
$= a^2 + b^2 = 4 + 2ab$
$a^2 + b^2 = 4 + 2 \times 8 = 20$

162. (b) $(a^2 + b^2)^3 = (a^3 + b^3)^2$

$\Rightarrow a^6 + b^6 + 3a^2b^2(a^2 + b^2) = a^6 + b^6 + 2a^3b^3$

$\Rightarrow a^6 + b^6 + 3a^4b^2 + 3a^2b^4 = a^6 + b^6 + 2a^3b^3$

$\Rightarrow 3a^4b^2 + 3a^2b^4 = 2a^3b^3$

$\Rightarrow a^2b^2 (3a^2 + 3b^2) = 2a^3b^3$

$3a^2 + 3b^2 = 2ab$

$3 (a^2 + b^2) = 2ab$

$\Rightarrow \frac{a^2+b^2}{ab} = \frac{2}{3}$

$\Rightarrow \frac{a}{b}+\frac{b}{a} = \frac{2}{3}$

163. (a) $x + \frac{1}{x} = 5 \quad x^2 + \frac{1}{x^2} + 2 = 25$

$x^2 + \frac{1}{x^2} = 23$

now, $\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$

divided by x^2,

$= \frac{\frac{x^4}{x^2}+\frac{3x^3}{x^2}+\frac{5x^2}{x^2}+\frac{3x}{x^2}+\frac{1}{x^2}}{\frac{x^4}{x^2}+\frac{1}{x^2}}$

$= \frac{x^2+3x+5+\frac{3}{x}+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}$

$= \frac{x^2+\frac{1}{x^2}+3\left(x+\frac{1}{x}\right)+5}{x^2+\frac{1}{x^2}}$

$= \frac{23+3(5)+5}{23} = \frac{43}{23}$

164. (b) $x^3 + \frac{1}{x^3} = 0$

$\left(x+\frac{1}{x}\right)^3 - 3x \times \frac{1}{x}\left(x+\frac{1}{x}\right) = 0$

$\left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) = 0$

$\left(x+\frac{1}{x}\right)^3 = 3\left(x+\frac{1}{x}\right)$

$\left(x+\frac{1}{x}\right)^2 = 3$

$\left(\left(x+\frac{1}{x}\right)^2\right)^2 = (3)^2$

$\left(x+\frac{1}{x}\right)^4 = 9$

165. (c) $x + \frac{1}{x} = 3$

(Squaring both sides)

$x^2 + \frac{1}{x^2} = 7$

On cubing both sides

$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=27$

$\Rightarrow x^3+\frac{1}{x^3}+3\times 3=27$

$x^3+\frac{1}{x^3}=18$

$\therefore \left(x^3+\frac{1}{x^3}\right)\left(x^2+\frac{1}{x^2}\right)=18\times 7$

$\left(x^5+\frac{1}{x^5}\right)+\left(x+\frac{1}{x}\right)=126$

$\left(x^5+\frac{1}{x^5}\right)+3=126$

$\left(x^5+\frac{1}{x^5}\right)=123$

166. (b) $x - \frac{1}{x} = 3$

$x^3 - \frac{1}{x^3} - 3\left(x-\frac{1}{x}\right) = 27$

$x^3 - \frac{1}{x^3} = 27 + 3 \times 3$

$= 27 + 9 = 36$

167. (a) $m^4 + \frac{1}{m^4} = 119$

$m^4 + \frac{1}{m^4} + 2 = 119 + 2$

$\left(m^2 + \frac{1}{m^2}\right)^2 = 121$

$m^2 + \frac{1}{m^2} = 11$

$m^2 + \frac{1}{m^2} - 2 = 11 - 2$

$\left(m-\frac{1}{m}\right)^2 = 9$

$m - \frac{1}{m} = \pm 3$

168. (d) $x + y + z = 6$

$(x - 1)^3 + (y - 2)^3 + (z - 3)^3$

$\therefore$ as $x + y + z = 6$

Take values

$x = 1, \; y = 2, \; z = 3$

$(1 + 2 + 3) = 6$

$\therefore (1 - 1)^3 + (2 - 2)^3 + (3 - 3)^2 = 0$

Now assume values in options. option 'd' satisfies the given relation.

Hence 'd' is correct.

169. (b) $a + b + c = 6$

$= a^2 + b^2 + c^2 = 14$

$a^3 + b^3 + c^3 = 36$

Put values as

$a = 1, \quad b = 2, \quad c = 3$

$1 + 2 + 3 = 6$

$1 + 4 + 9 = 14$

$1 + 8 + 27 = 36$

$\therefore \quad abc = 1 \times 2 \times 3 = 6$

Alternate:-

$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$36 = 14 + 2(ab + bc + ca)$

$(ab + bc + ca) = 11$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow 36 - 3abc = 6 (14 - 11)$

$\Rightarrow 36 - 3abc = 6 \times 3$

$-3abc = 18 - 36$

$3abc = 18$

$abc = 6$

170. (a) $a + b = 1$

By cubing

$a^3+b^3 +3ab (a + b) = 1^3$

$a^3 + b^3 + 3ab = 1 (a + b=1)$

$a^3 + b^3 + 3ab = k$

From above both equations

$k = 1$

171. (d) $a = 34, b = 33, c = 33$

$\because a^3 + b^3 + c^3 - 3abc = (a + b + c)$

$\times \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$= (34 + 33 + 33) \times \frac{1}{2}$

$\left[(34-33)^2+(33-33)^2+(33-34)^2\right]$

$= 100 \times \frac{1}{2}(1 + 0 + 1)$

$= 100 \times 1 = 100$

172. (a) $2^x.2^y = 8 \Rightarrow 2^{x+y} = 2^3$

$x + y = 3$ .....(i)

$9^x.3^y = 81$

$3^{2x}.3^y = 3^4$

$\Rightarrow 2x + y = 4$ ....(ii)

From equation (i) and (ii),

$$\begin{array}{r} x+y=3 \\ \underline{2x+y=4} \\ \underline{-x=-1} \end{array}$$

$x = 1$

$y = 2$

$\therefore (x, y) = (1, 2)$

173. (b) $2x + y = 5$ ....(i)

$x + 2y = 4$ ...(ii)

Multiply equation (ii) by 2.

$2x + 4y = 8$ .....(iii)

Now substracting equation (i) from (iii)

$$\begin{array}{r} 2x + 4y = 8 \\ \underline{2x + y = 5} \\ 3y = 3 \end{array}$$

$y = 1$

$x = 2$

$\therefore$ intersection point = (2, 1)

174. (a) $y : x = 4 : 15$

$\therefore \frac{y}{x} = \frac{4}{15}$

$\therefore \frac{x-y}{x+y} \Rightarrow \frac{x\left(1-\frac{y}{x}\right)}{x\left(1+\frac{y}{x}\right)}$ taking x common.

$\Rightarrow \frac{1-\frac{4}{15}}{1+\frac{4}{15}} \Rightarrow \frac{11}{15} \times \frac{15}{19} = \frac{11}{19}$

175. (b) $x^4 - 2x^2 + k$

$\Rightarrow (A + B)^2 = A^2 + 2AB + B^2$

$(A - B)^2 = A^2 - 2AB + B^2$

$\Rightarrow (x^2)^2 - 2 \times x^2 + k$

$(A)^2 - 2 \times AB + B^2$

$\therefore A = x^2, B = -1$

$B^2 = K$

$(-1)^2 = K$

$K = 1$

176. (c) $(x - 3)^2 + (y - 5)^2 + (z - 4)^2 = 0$

$\therefore (x - 3)^2 = 0$ $x = 3$

$(y - 5)^2 = 0$ $y = 5$

$(z - 4)^2 = 0$ $z = 4$

$\therefore \frac{x^2}{9} + \frac{y^2}{25} + \frac{c^2}{16}$

$\Rightarrow \frac{9}{9} + \frac{25}{25} + \frac{16}{16} = 3$

177. (d) $x \propto \frac{1}{y^2}$

(Inversely proportional)

$x = \frac{k}{y^2}$

(y = 2) for (x = 1) (Given)

$\therefore 1 = \frac{k}{(2)^2} \Rightarrow 1 = \frac{k}{4}$

$k = 4$

$\therefore$ For y = 6

$x = \frac{4}{(6)^2} = \frac{1}{9} = \frac{1}{9}$

178. (d) $a^2 + b^2 + c^2 + 3 = 2(a - b - c)$

$a^2 + b^2 + c^2 + 3 - 2a + 2b + 2c = 0$

$a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$

$(a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$a = 1, b = -1, c = -1$

$\therefore 2a - b + c = 2 + 1 - 1 = 2$

179. (d) $x^2 - y^2 = 80$

$\Rightarrow (x - y)(x + y) = 80$

$x - y = 8$ .....(I)

$\therefore (x + y) \times 8 = 80$

$(x + y) = 10$ .....(II)

Now average of x and y

$= \frac{x+y}{2} = \frac{10}{2} = 5$

180. (a) Third proportional of a and b

$= \frac{b^2}{a}$

Third proportion of $\left(\frac{x}{y} + \frac{y}{x}\right)$ and $\sqrt{x^2 + y^2}$

$\frac{\left(\sqrt{x^2+y^2}\right)^2}{\frac{x}{y}+\frac{y}{x}} = \frac{x^2+y^2}{\frac{x^2+y^2}{xy}} = xy$

181. (c) $\frac{4x}{3} + 2P = 12$, $x = 6$ (Given)

$\frac{4 \times 6}{3} + 2P = 12$

$2P = 12 - 8$

$P = \frac{4}{2}$

$P = 2$

182. (a) $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}$

(By Rationalization of denominator)

$\frac{4+3\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$= \frac{(4+3\sqrt{3})(7-4\sqrt{3})}{49-48}$

$$\begin{array}{r} 4 + 3\sqrt{3} \\ \times\ 7 - 4\sqrt{3} \\ \hline -16\sqrt{3} - 12 \times 3 \\ 28 + 21\sqrt{3} \\ \hline 28 - 36 + 5\sqrt{3} = 5\sqrt{3} - 8 \end{array}$$

183. (d) $a = \sqrt{6} - \sqrt{5}$

$b = \sqrt{5} - 2$

$c = 2 - \sqrt{3}$

Rationalize all the terms

$a = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \sqrt{6} + \sqrt{5}$

$= \frac{1}{\sqrt{6}+\sqrt{5}}$

$b = \frac{\sqrt{5}-2}{\sqrt{5}+2} \times \sqrt{5} + 2 = \frac{1}{\sqrt{5}+2}$

$= \frac{1}{\sqrt{5}+\sqrt{4}}$

$c = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times 2 + \sqrt{3}$

$= \frac{1}{2+\sqrt{3}} = \frac{1}{\sqrt{4}+\sqrt{3}}$

Now, $a = \frac{1}{\sqrt{6}+\sqrt{5}}$

$b = \frac{1}{\sqrt{5}+\sqrt{4}}$

$c = \frac{1}{\sqrt{4}+\sqrt{3}}$

Now the term whose denominator is largest is the smallest term.

So, a < b < c.

184. (b) $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$

$\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}} = ?$

$\therefore x = \frac{4 \times \sqrt{5} \times \sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2 \times \sqrt{5} \times \sqrt{12}}{\sqrt{5}+\sqrt{3}}$

or $\frac{2 \times \sqrt{20} \times \sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$\frac{x}{\sqrt{12}} = \frac{2\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

by C – D rule

$\frac{x+\sqrt{12}}{x-\sqrt{12}} = \frac{2\sqrt{5}+\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{5}-\sqrt{3}}$

$= \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$\frac{x}{\sqrt{20}} = \frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$\frac{x+\sqrt{20}}{x-\sqrt{20}} = \frac{2\sqrt{3}+\sqrt{5}+\sqrt{3}}{2\sqrt{3}-\sqrt{5}-\sqrt{3}}$

$= \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$

$\therefore \frac{x+\sqrt{12}}{x-\sqrt{12}} + \frac{x+\sqrt{20}}{x-\sqrt{20}}$

$= \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$

$= \frac{3\sqrt{5}+\sqrt{3}-3\sqrt{3}-\sqrt{5}}{\sqrt{5}-\sqrt{3}}$

$= \frac{2\sqrt{5}-2\sqrt{3}}{\sqrt{5}-\sqrt{3}} = 2$

185. (b) $x = 5 - \sqrt{21}$

$2x = 10 - 2\sqrt{21}$ .....(i)

$\Rightarrow 2x = \left(\sqrt{7}\right)^2 + \left(\sqrt{3}\right)^2 - 2\left(\sqrt{7}\right)\left(\sqrt{3}\right)$

$\Rightarrow 2x = \left(\sqrt{7}-\sqrt{3}\right)^2$

$\Rightarrow x = \frac{1}{2}\left(\sqrt{7}-\sqrt{3}\right)^2$

$$\Rightarrow \sqrt{x} = \frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{7}-\sqrt{3}\right)^2}$$

$$= \frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$$

$$\therefore \frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{32-\left(10-2\sqrt{21}\right)}-\sqrt{21}\right)}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{22+2\sqrt{21}}-\sqrt{21}\right)}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{\left(\sqrt{21}+1\right)^2}-\sqrt{21}\right)}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{21}+1-\sqrt{21}\right)}$$

$$= \frac{\left(\sqrt{7}-\sqrt{3}\right)}{\sqrt{2}}$$

186. (b) $6x - 5y = 13$ ....(I)

$7x + 2y = 23$ ....(II)

Multiply equation (I) with 7 and (II) with 6

$\therefore$ $7(6x - 5y = 13)$

$42x - 35y = 91$ .....(III)

$42x + 12y = 138$ .....(IV)

Subtract (IV) from (III)

$$\begin{array}{r} 42x - 35y = 91 \\ \underline{42x + 12y = 138} \\ -47y = -47 \\ \hline y = 1 \end{array}$$

$\therefore$ $x = 3$

$\therefore$ $11x + 18y$

$= 11 \times 3 + 18 \times 1 = 33 + 18 = 51$

187. (a) $(x^{b+c})^{b-c}\ (x^{c+a})^{c-a}\ (x^{a+b})^{a-b}\ (x \neq 0)$

$= x^{b^2-c^2} \cdot x^{c^2-a^2} \cdot x^{a^2-b^2}$

$x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1$

188. (c) $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$

$\frac{x}{a} - \frac{1}{a} = -\frac{1}{x}$

$\left(\frac{x-1}{a}\right) = -\frac{1}{x}$

$\frac{1-x}{a} = \frac{1}{x}$

$x(1 - x) = a$

$x - x^2 = a$

189. (c) $x + \frac{1}{x} = 99$

$\therefore$ $x^2 + 1 = 99x$

$2(x^2 + 1) = 2 \times 99x$

$2x^2 + 2 = 198x$

$= \frac{100x}{2x^2 + 2 + 102x}$

$= \frac{100x}{198x + 102x} = \frac{100x}{300x} = \frac{1}{3}$

190. (c) $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$

$\Rightarrow \frac{4x}{x} - \frac{3}{x} + \frac{4y}{y} - \frac{3}{y} + \frac{4z}{z} - \frac{3}{z} = 0$

$= 4 - \frac{3}{x} + 4 - \frac{3}{y} + 4 - \frac{3}{z} = 0$

$12 - 3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = 0$

$-3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = -12$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 4$

191. (c) $\frac{xy}{x+y} = a$, $\frac{xz}{x+z} = b$, $\frac{yz}{y+z} = c$

Now

$\Rightarrow \frac{x+y}{xy} = \frac{1}{a}$,

$\frac{x+z}{xz} = \frac{1}{b}$, $\frac{y+z}{yz} = \frac{1}{c}$

$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{a}$, $\frac{1}{z} + \frac{1}{x} = \frac{1}{b}$,

$\frac{1}{z} + \frac{1}{y} = \frac{1}{c}$

Now we have to find the value of x

$\therefore \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{1}{y} + \frac{1}{x} + \frac{1}{z}$

$+\frac{1}{x} - \frac{1}{y} - \frac{1}{z}$

$\therefore \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{2}{x}$

$\frac{bc + ac - ab}{abc} = \frac{2}{x}$

$x = \frac{2abc}{bc + ac - ab}$

192. (d) $xy = 8$ Given

So, $(x, y) = (1, 8)$

We have to question the options and check them

(8, 1)

(2, 4)

(4, 2)

$\therefore$ $2x + y = 2 \times 1 + 8 = 10$

$2 \times 8 + 1 = 11$

$2 \times 2 + 4 = 8$ minimum

$2 \times 4 + 2 = 10$

Hence in this question we have all the options. So, take all positive factor otherwise we should have to take –ve values also.

$(x, y) = (1, 8)$

(8, 1)

(2, 4)

(4, 2)

(–1, –8)

(–8, –1)

(–2, –4)

(–4, –2)

193. (b) $x^2 + x + 1 = 0$ .....(i)

$\left(x+\frac{1}{2}\right)^2 + q^2 = 0$

$\Rightarrow x^2 + \frac{1}{4} + 2 \times \frac{1}{2} \times x + q^2$

$= x^2 + x + \left(q^2 + \frac{1}{4}\right) = 0$ .....(ii)

Comparing constant term of equation (i) and (ii)

$\therefore$ $q^2 + \frac{1}{4} = 1$

$=$ $q^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$q = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

194. (b) $a^2 - 4a - 1 = 0$

$a^2 - 1 = 4a$

$a - \frac{1}{a} = 4$

Squaring both sides

$a^2 + \frac{1}{a^2} - 2 = 16$

$a^2 + \frac{1}{a^2} = 18$

$\therefore$ $a^2 + \frac{1}{a^2} + 3\left(a - \frac{1}{a}\right)$

$= 18 + 3 \times 4 = 18 + 12 = 30$

195. (d) $(x - 2)(x - 9)$

$= x^2 - 9x - 2x + 18$

$= x^2 - 11x + 18$

$= ax^2 + bx + c = 0$

For minimum value

$\frac{4ac - b^2}{4a}$

$= \frac{4\times1\times18-(-11)^2}{4\times1} = \frac{72-121}{4} = \frac{-49}{4}$

196. (d) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$4x\left(\sqrt{3}x+2\right) - \sqrt{3}\left(\sqrt{3}x+2\right) = 0$

$\left(4x-\sqrt{3}\right)\left(\sqrt{3}x+2\right) = 0$

197. (c) $\sqrt{x} = \sqrt{3} - \sqrt{5}$

Squaring both sides

$x = 3 + 5 - 2\sqrt{3}\sqrt{5}$

$x = 8 - 2\sqrt{5} \times \sqrt{3}$

$x - 8 = -2\sqrt{15}$

(Squaring both sides)

$x^2 + 64 - 16x = 60$

$x^2 + 4 - 16x = 0$

$x^2 + 6 - 16x = 2$

198. (b) $x^2 = y + z$

$y^2 = z + x$

$z^2 = x + y$

$\Rightarrow x^2 + x = x + y + z$

Adding x on both sides

$x(x+1) = x + y + z$

$\frac{1}{(x+1)} = \frac{x}{x+y+z}$

Similarly

$\frac{1}{y+1} = \frac{y}{x+y+z}$ and $\frac{1}{z+1}$

$= \frac{z}{x+y+z}$

$\therefore \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{1+z}$

$= \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z}$

$= \frac{x+y+z}{x+y+z} = 1$

199. (c) $a + b + c = 0$

Have values

$a = 1$

$b = 2$

$c = -3$

$\Rightarrow \left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$

$\Rightarrow \left(\frac{1+2}{-3} + \frac{2-3}{1} + \frac{-3+1}{2}\right)\left(\frac{1}{2-3} + \frac{2}{-3+1} + \frac{-3}{1+2}\right)$

$\Rightarrow (-1-1-1)(-1-1-1)$

$\Rightarrow -3 \times -3 = 9$

200. (a) $a + \frac{1}{b} = 1$, $b + \frac{1}{c} = 1$

Values of a, b, c assume

$a = \frac{1}{2}$

$b = 2$

$c = -1$

$\therefore abc = \frac{1}{2} \times 2 \times -1 = -1$

201. (c) $a + b + c = 2s$

$\therefore$ let

$a = 2$

$b = 1$

$c = 1$

$s = 2$

$\therefore \frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2+b^2+c^2}$

$\Rightarrow \frac{(2-2)^2 + (2-1)^2 + (2-1)^2 + 2^2}{2^2+1^2+1^2}$

$\Rightarrow \frac{0+1+1+4}{4+1+1} = \frac{6}{6} = 1$

202. (d) $x = 3 + 2\sqrt{2}$

$x^2 = (3 + 2\sqrt{2})^2$

(Squaring both sides)

$= 9 + 8 + 12\sqrt{2} = 17 + 12\sqrt{2}$

$\frac{1}{x^2} = \frac{1}{17+12\sqrt{2}} \times \frac{17-12\sqrt{2}}{17-12\sqrt{2}}$

$= 17 - 12\sqrt{2}$

$\therefore \frac{1}{x^2} + x^2$

$= 17 + 12\sqrt{2} + 17 - 12\sqrt{2} = 34$

203. (b) $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$

$\Rightarrow 3x - 2 = \frac{3}{x}$

$\Rightarrow 3x - \frac{3}{x} = 2$

$\Rightarrow 3\left(x - \frac{1}{x}\right) = 2$

$\Rightarrow x - \frac{1}{x} = \frac{2}{3}$

Squaring both sides

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = \frac{4}{9}$

$\Rightarrow x^2 + \frac{1}{x^2} = \frac{4}{9} + 2 = 2\frac{4}{9}$

204. (a) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

$x + \frac{1}{x} = 3$

Squaring both sides

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$

$x^2 + \frac{1}{x^2} = 7$

$\therefore x^2 + \frac{1}{x^2} + x + \frac{1}{x}$

$= 7 + 3 = 10$

205. (c) $a^2 + b^2 = 5ab$

$\Rightarrow \frac{a^2}{ab} + \frac{b^2}{ab} = 5$

$\Rightarrow \frac{a}{b} + \frac{b}{a} = 5$

Squaring both sides

$\left(\frac{a}{b}\right)^2 + \left(\frac{b}{a}\right)^2 + 2 \times \frac{a}{b} \times \frac{b}{a} = 25$

$\frac{a^2}{b^2} + \frac{b^2}{a^2} = 25 - 2 = \mathbf{23}$

206. (d) $xy + yz + zx = 0$

$\therefore xy + zx = -yz$

$\Rightarrow xy + yz = -zx$

$\Rightarrow yz + zx = -xy$

$\therefore \frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy}$

Putting values of $-yz$, $-zx$, $-xy$ from above

$\Rightarrow \frac{1}{x^2 + (xy+zx)} + \frac{1}{y^2 + (xy+yx)} + \frac{1}{z^2 + (yz+zx)}$

$\Rightarrow \frac{1}{x(x+y+z)} + \frac{1}{y(x+y+z)} + \frac{1}{z(x+y+z)}$

$\Rightarrow \frac{1}{(x+y+z)}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

$\Rightarrow \frac{1}{(x+y+z)}\left(\frac{zy+xz+xy}{xyz}\right)$

$\Rightarrow \frac{1}{x+y+z} \times 0 = 0$

207. (c) $a + b + c = 9$

For minimum value $a = b = c$

$\Rightarrow 3a = 9$

$a = \frac{9}{3} = 3$

For minimum value $a = b = c = 3$

$a^2 + b^2 + c^2 = 3^2 + 3^2 + 3^2$

$\Rightarrow 9+9+9 \Rightarrow 27$

208. (d) $a^2 + b^2 + 4c^2 = 2(a + b - 2c) - 3$

$\Rightarrow a^2 + b^2 + 4c^2 - 2a - 2b + 4c + 3 = 0$

$\Rightarrow a^2 - 2a + 1 + b^2 - 2b + 1 + 4c^2 + 4c + 1 = 0$

$(a-1)^2 + (b-1)^2 + (2c+1)^2 = 0$

$\therefore a - 1 = 0$ $\quad a = 1$

$b - 1 = 0$ $\quad b = 1$

$2c + 1 = 0$ $\quad c = \frac{-1}{2}$

$\therefore a^2 + b^2 + c^2$

$= 1 + 1 + \frac{1}{4} = 2 + \frac{1}{4} = \frac{9}{4} = 2\frac{1}{4}$

209. (a) $\frac{x-a^2}{b+c} + \frac{x-b^2}{c+a} + \frac{x-c^2}{a+b}$

$= 4(a + b + c)$

Note:- In such type of question to save your valuable time assume values as per your need which make your calculation easier.

assume a = 1, b = 0, c = 1

Make sure there will be no $\left(\frac{0}{0}\right)$ form

$\therefore \quad \frac{x-1}{1+0} + \frac{x-0}{1+1} + \frac{x-1}{1+0} = 4$

$(1 + 0 + 1)$

$\Rightarrow \quad x - 1 + \frac{x}{2} + x - 1 = 4 \times 2$

$x + \frac{x}{2} + x = 8 + 2$

$\frac{5x}{2} = 10$

$x = 4$

Now put values in options take option (a), $(a + b + c)^2 = (1 + 0 + 1)^2 = 4$

210. (b) $4x - y = 2$

$2x - 8y + 4 = 0$

Note:- For two linear equaltions

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where x and y are variable.

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then there will be unique solution.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then infinite solution

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then no solution.

$\therefore \quad \frac{4}{2} \neq \frac{-1}{-8}$

So, the equations have only one solution

211. (d) $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$

$\Rightarrow \quad \frac{a}{b} \times \frac{b}{c} = \frac{4}{5} \times \frac{15}{16} = \frac{3}{4}$

$\therefore \quad \frac{a}{c} = \frac{3}{4}$

$\therefore \quad \frac{18c^2-7a^2}{45c^2+20a^2} = \frac{c^2\left(18-7\frac{a^2}{c^2}\right)}{c^2\left(45+20\frac{a^2}{c^2}\right)}$

$= \frac{18-7\left(\frac{a}{c}\right)^2}{45+20\left(\frac{a}{c}\right)^2} = \frac{18-7\times\frac{9}{16}}{45+20\times\frac{9}{16}}$

$= \frac{18-\frac{63}{16}}{45+\frac{45}{4}} = \frac{225\times4}{16\times225} = \frac{1}{4}$

212. (d) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$

Go through options 'd'

take $x = y = z$

$\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2} = \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2}$

$\therefore$ Option d is right

213. (b) $a + b = 1$

$c + d = 1$

$a - b = \frac{d}{c}$

or $\frac{1}{a-b} = \frac{c}{d}$

$\Rightarrow \quad \frac{a+b}{a-b} = \frac{c}{d} \qquad (\therefore a + b = 1)$

by C & D rule

$\Rightarrow \quad \frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{c+d}{c-d}$

$\Rightarrow \quad \frac{2a}{2b} = \frac{c+d}{c-d}$

$\Rightarrow \quad \frac{a}{b} = \frac{c+d}{c-d}$

Now multiply & divide by (c + d)

$\frac{a}{b} = \frac{(c+d)}{(c-d)} \times \frac{(c+d)}{(c+d)} = \frac{(c+d)^2}{c^2-d^2}$

$\Rightarrow \quad \frac{a}{b} = \frac{(c+d)^2}{(c^2-d^2)}$

$c + d = 1$

$\frac{a}{b} = \frac{1}{c^2-d^2}$

$c^2 - d^2 = \frac{b}{a}$

214. (b) $x = 3t, \qquad .....(I)$

$y = \frac{1}{2}(t + 1)$

$x = 2y$

$\Rightarrow \quad x = 2\times\frac{1}{2}(t + 1)$

$x = t + 1 \qquad .....(II)$

$\therefore \quad 3t = t + 1$

(from equation (i) and (ii)

$2t = 1$

$t = \frac{1}{2}$

215. (c) $x^2 + \frac{1}{5}x + a^2$

$A^2 + 2 \times AB + B^2 = (A + B)^2$

$x^2 + 2\times\frac{1}{10}\times x + a^2 = \left(x + \frac{1}{10}\right)^2$

$A = x$

$B = \frac{1}{10}$

$B = a = \frac{1}{10}$

216. (d) $2 - 3x - 4x^2 = 0$

$-4x^2 - 3x + 2 = 0$

$ax^2 + bx + c = 0$

In quadratic equation

(i) When $a > 0$

Minimum value $= \frac{4ac-b^2}{4a}$

(ii) When $a < 0$

Maximum value $= \frac{4ac-b^2}{4a}$

$\therefore$ In $-4x^2-3x + 2$

$a < 0$

$\therefore$ Maximum value

$=$ Maximum value

$\frac{4\times-4\times2-(-3)^2}{4\times-4}$

$= \frac{-32-9}{-16} = \frac{41}{16}$

217. (b) If $(x - 1)$ and $(x + 3)$ are the factors of $x^2 + k_1x + k_2$

$\therefore$ When $(x - 1) = 0$

$x = 1$

$x^2 + k_1x + k_2 = 0$

$1 + k_1 + k_2 = 0$

$k_1 + k_2 = -1 \qquad .....(I)$

When $(x + 3) = 0$

$x = -3$

$9 - 3k_1 + k_2 = 0$

$-3k_1 + k_2 = -9 \qquad .....(II)$

From equation (I) and (II)

$k_1 = 2, K_2 = -3$

218. (d) $\frac{5x}{2x^2+5x+1} = \frac{1}{3}$

$\frac{5}{\frac{2x^2}{x}+\frac{5x}{x}+\frac{1}{x}} = \frac{1}{3}$

$\frac{5}{2x+\frac{1}{x}+5} = \frac{1}{3}$

$\left(2x+\frac{1}{x}+5\right) = 15$

$2x + \frac{1}{x} = 10$

Divide by 2 both sides

$x + \frac{1}{2x} = \frac{10}{2} = 5$

219. (c) $x^2 + \frac{1}{x^2} = 83$

Subtracting 2 from both sides

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 83 - 2$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 81$

$\Rightarrow x - \frac{1}{x} = 9$

Take cube on both sides

$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 729$

$x^3 - \frac{1}{x^3} - 3 \times 9 = 729$

$x^3 - \frac{1}{x^3} = 729 + 27 = 756$

220. (d) $\left(a + \frac{1}{a}\right)^2 = 3$

$a + \frac{1}{a} = \sqrt{3}$

Take cube on both sides

$\left(a + \frac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$

$= a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = 3\sqrt{3}$

$= a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$= a^3 + \frac{1}{a^3} = 0$

221. (b) $\frac{x}{x^2 - 2x + 1} = \frac{1}{3}$

$\Rightarrow \frac{1}{\frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x}} = \frac{1}{3}$

$\Rightarrow \frac{1}{x + \frac{1}{x} - 2} = \frac{1}{3}$

$x + \frac{1}{x} - 2 = 3$

$\Rightarrow x + \frac{1}{x} = 3 + 2 = 5$

Taking cube on both sides

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (5)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 5 = 125$

$\Rightarrow x^3 + \frac{1}{x^3} = 110$

222. (b) $x + \frac{1}{x} = 4$

Squaring both sides

$x^2 + \frac{1}{x^2} + 2 = 16$

$x^2 + \frac{1}{x^2} = 14$

Squaring again

$x^4 + \frac{1}{x^4} = 196 - 2 = 194$

223. (c) $x + y + z = 6$

$x^2 + y^2 + z^2 = 20$

$\Rightarrow (x + y + z)^2 = (6)^2$

$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 36$

$\Rightarrow 20 + 2(xy + yz + zx) = 36$

$\Rightarrow 2(xy + yz + zx) = 16$

$xy + yz + zx = 8$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - zx - yz)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 6(20 - 8)$

$= 6 \times 12 = 72$

224. (b) $x = 1 - \sqrt{2}$

$\frac{1}{x} = \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$

$= \frac{1 + \sqrt{2}}{1 - 2} = \frac{1 + \sqrt{2}}{-1} = -\left(\sqrt{2} + 1\right)$

$\therefore x - \frac{1}{x} = 1 - \sqrt{2} + \sqrt{2} + 1 = 2$

Take cube

$\left(x - \frac{1}{x}\right)^3 = (2)^3 \Rightarrow \left(x - \frac{1}{x}\right)^3 = 8$

225. (b) $x = a - b$

$y = b - c$

$z = c - a$

$x + y + z = a - b + b - c + c - a = 0$

$\therefore x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = 0$

226. (d) $x = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

$y = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

$\therefore x = \frac{1}{y}$

$y = \frac{1}{x} \Rightarrow xy = 1$

$\therefore x + y = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} + \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

$\Rightarrow \frac{\left(\sqrt{3} - \sqrt{2}\right)^2 + \left(\sqrt{3} + \sqrt{2}\right)^2}{1}$

$\Rightarrow \frac{3 + 2 - 2\sqrt{6} + 3 + 2 + 2\sqrt{6}}{1} = 10$

$\Rightarrow (x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow (10)^3 = x^3 + y^3 + 3 \times 1(10)$

$\Rightarrow x^3 + y^3 = 1000 - 30 = 970$

227. (d) $(x - a)(x - b) = 1$

$(x - a) = \frac{1}{(x - b)}$

$\Rightarrow a - b = -5$ (Given)

add and subtract x

$a - b + x - x = -5$

$(a - x) + (x - b) = -5$

$(x - b) - (x - a) = -5$

$(x - a) - (x - b) = +5$

$\therefore (x - a) - \frac{1}{(x - a)} = +5$

Taking cube on both sides

$(x - a)^3 - \frac{1}{(x - a)^3} - 3(x - a) \times \frac{1}{(x - a)}\left((x - a) - \frac{1}{(x - a)}\right) = (5)^3$

$\Rightarrow (x - a)^3 - \frac{1}{(x - a)^3} - 3 \times (5) = 125$

$(x - a)^3 - \frac{1}{(x - a)^3} = 125 + 15 = 140$

228. (a) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$

$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$x \quad y \quad z$

$\Rightarrow a - 1 = 0 \qquad a = 1$

$\Rightarrow b + 1 = 0 \qquad b = -1$

$\Rightarrow c + 1 = 0 \qquad c = -1$

$= 4 \times 1 - 3 \times -1 + 5 \times -1$

$= 4 + 3 - 5 = 2$

229. (c) $2x + \frac{2}{x} = 3$

$\Rightarrow x + \frac{1}{x} = \frac{3}{2}$

Taking cube on both sides

$= \left(x + \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$

$x + \frac{1}{x} = a$

$x^3 + \frac{1}{x^3} = a^3 - 3a$

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{3}{2} = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2}$

$= \frac{27 - 36}{8} = \frac{-9}{8}$

$\therefore x^3 + \frac{1}{x^3} + 2 = \frac{-9}{8} + 2$

$= x^3 + \frac{1}{x^3} + 2 = \frac{-9 + 16}{8} = \frac{7}{8}$

230. (b) $a + b + c = 15$

$a^2 + b^2 + c^2 = 83$

$\therefore \quad (a + b + c)^2 = (15)^2$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 225$

$\Rightarrow \quad 83 + 2(ab + bc + ca) = 225$

$\Rightarrow \quad 2(ab + bc + ca) = 225 - 83 = 142$

$ab + bc + ca = 71$

$\Rightarrow \quad a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow \quad a^3 + b^3 + c^3 - 3abc = 15(83 - 71) = 15 \times 12 = 180$

231. (a) $a - b = 3$

$a^3 - b^3 = 27$

For exams assume the values of a>b

Let $a = 3, b = 0$

$a - b = 3 - 0 = 3$

and $(3)^3 - 0 = 27$

Hence, $a + b = 3 + 0 = 3$

232. (b) $x + \frac{1}{x+1} = 1$

adding (1) both sides

$\therefore \quad x + 1 + \frac{1}{x+1} = 1 + 1$

$\Rightarrow \quad (x+1) + \left(\frac{1}{x+1}\right) = 2$

Put $x + 1 = 1$

and $\frac{1}{x+1} = 1$

$\therefore \quad (x + 1)^5 + \frac{1}{(x+1)^5}$

$= 1 + 1 = 2$

233. (d) If $a + b + c = 0$

then, $a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$

234. (c) $x = y = 333, z = 334$

$\Rightarrow \quad x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)((x - y)^2 + (y - z)^2) + (z - x)^2)$

$\Rightarrow \quad \frac{1}{2}(333 + 333 + 334)[(333 - 333)^2 + (333 - 334)^2 + (334 - 333)^2]$

$\Rightarrow \quad \frac{1}{2}(1000)(0 + 1 + 1) = 1000$

235. (a) $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

Let $a^2 - b^2 = A$

$b^2 - c^2 = B$

$c^2 - a^2 = C$

$A + B + C = (a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

$\because \quad A + B + C = 0$

$\therefore \quad A^3 + B^3 + C^3 - 3ABC$

$= \frac{1}{2}(A + B + C)[(A - B)^2 + (B - C)^2 + (C - A)^2]$

$A^3 + B^3 + C^3 - 3ABC = 0$

$A^3 + B^3 + C^3 = 3ABC$

Where $A = a^2 - b^2$ etc.

$\therefore \quad A^3 + B^3 + C^3 = 3 \times (a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

Hence $(a + b)(a - b)$ is a factor

236. (b) $a = \frac{b^2}{b-a}$

$\Rightarrow \quad a(b - a) = b^2$

$ab - a^2 = b^2$

$a^2 + b^2 - ab = 0$

$\Rightarrow \quad a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$\therefore \quad a^3 + b^3 = 0$

237. (b) $p - 2q = 4$

Take cube on both sides

$(p - 2q)^3 = (4)^3$

$p^3 - 8q^3 - 3p \times 2q(p - 2q) = 64$

$p^3 - 8q^3 - 6pq \times 4 = 64$

$p^3 - 8q^3 - 24pq = 64$

$p^3 - 8q^3 - 24pq - 64 = 0$

238. (c) $x = -1$

$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x} - 1$

$= \frac{1}{(-1)^{99}} + \frac{1}{(-1)^{98}} + \frac{1}{(-1)^{97}} + \frac{1}{(-1)^{96}} + \frac{1}{(-1)^{95}} + \frac{1}{(-1)^{94}} + \frac{1}{(-1)} - 1$

$= -1 + 1 - 1 + 1 - 1 + 1 + \frac{1}{-1} - 1 = -2$

239. (a) $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$

$\Rightarrow \quad \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = \frac{1}{\left(2^{\frac{1}{3}}\right)^2 + 2^{\frac{1}{3}} + (1)^2}$

$\Rightarrow (\therefore A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$\therefore \quad$ Put, $A = 2^{\frac{1}{3}}, \quad B = 1$

$= \frac{\left(2^{\frac{1}{3}} - 1\right)}{\left(2^{\frac{1}{3}} - 1\right)\left(\left(2^{\frac{1}{3}}\right)^2 + 2^{\frac{1}{3}} + (1)^2\right)}$

$= \frac{\left(2^{\frac{1}{3}} - 1\right)}{\left(2^{\frac{1}{3}}\right)^3 - (1)^3} = \left(2^{\frac{1}{3}} - 1\right)$

$2^{\frac{1}{3}} - 1 = a\left(2^{\frac{2}{3}}\right) + b(2)^{\frac{1}{3}} + c$

(Comparing the terms)

$a = 0$

$b = 1$

$c = -1$

$\therefore \quad a + b + c = 0 + 1 - 1 = 0$

240. (d) $x = \sqrt[3]{2 + \sqrt{3}}$

$x^3 = 2 + \sqrt{3}$

$\frac{1}{x^3} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$

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$\Rightarrow \quad \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$

$\therefore \quad x^3 + \frac{1}{x^3} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

241. (d) $x = \sqrt[3]{5} + 2$

$\Rightarrow \quad x - 2 = \sqrt[3]{5}$

Take cube on both sides

$\Rightarrow \quad (x - 2)^3 = \left(\sqrt[3]{5}\right)^3$

$\Rightarrow \quad x^3 - 8 - 3 \times 2 \times x[x - 2] = 5$

$\Rightarrow \quad x^3 - 8 - 6x^2 + 12x = 5$

$\therefore \quad x^3 - 6x^2 + 12x - 13 = 0$

242. (b) $\frac{p^2 - p}{2p^3 + p^2} + \frac{p^2 - 1}{p^2 + 3p} + \frac{p^2}{p + 1}$

In such type of question assume values of p.

$\therefore \quad$ Let $p = 1$

$\therefore \quad \frac{1-1}{2+1} + \frac{1-1}{1+3} + \frac{1}{1+1}$

$= 0 + 0 + \frac{1}{2} = \frac{1}{2}$

Now check options (b)

$\frac{1}{2p^2} = \frac{1}{2}$

Hence option (b) is Answer.

243. (b) $x + \frac{1}{x} = 2$

$\Rightarrow$ Put x = 1

$\therefore 1 + \frac{1}{(1)} = 2$

2 = 2 (satisfy)

$\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$

= (1 + 1) (1 + 1) = 2 × 2 = 4

244. (a) a, b, c, are +ve integers

So, minimum value is $a=b=c=1$

$\therefore$ Putting the value of x in equation

$a^3 + b^3 + c^3 - 3abc$

= 1 + 1 + 1 – 3 × 1× 1× 1 =0

Hence minimum value is 0.

245. (d) $f(x) = 12x^3 - 13x^2 - 5x + 7$

If we divide $f(x)$ by

$\Rightarrow$ $(3x + 2)$ then

$3x + 2 = 0$

$x = -\frac{2}{3}$

$\therefore f\left(\frac{-2}{3}\right) = 12\left(\frac{-2}{3}\right)^3 - 13\left(\frac{-2}{3}\right)^2 - 5\left(\frac{-2}{3}\right) + 7$

$= -12\times\frac{8}{27} - \frac{52}{9} + \frac{10}{3} + 7$

$= \frac{-96-156+90+189}{27}$

$= \frac{-252+279}{27} = \frac{27}{27} = 1$

246. (b) $2x^2 - 7x + 12 = 0$

roots are α, β

$\therefore \alpha\beta = +\frac{c}{a}$, $\alpha + \beta = \frac{-b}{a}$

$\therefore \alpha+\beta = +\frac{7}{2}$, $\alpha\beta = \frac{12}{2} = 6$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{7}{2}\right)^2 - 2\times 6}{6} = \frac{\frac{49}{4} - 12}{6}$

$= \frac{49-48}{6\times 4} = \frac{1}{24}$

247. (c) $x^3 + \frac{3}{x} = 4(a^3 + b^3)$ ...(i)

$3x + \frac{1}{x^3} = 4(a^3 - b^3)$ ...(ii)

equation (i) + (ii)

$\left(x+\frac{1}{x}\right)^3 = 8a^3$

$x + \frac{1}{x} = 2a$ ...(iii)

Equation (i) – (ii)

$x - \frac{1}{x} = 2b$ ...(iv)

Equation (iii) – (iv)

$2(a - b) = \frac{2}{x}$

$a - b = \frac{1}{x}$

$a + b = x$

$\because a^2 - b^2 = (a + b) \times (a - b)$

$= x \times \frac{1}{x}$

$a^2 - b^2 = 1$

248. (c) $x = 6 + \frac{1}{x}$

$\Rightarrow x - \frac{1}{x} = 6$

Taking square on both sides

$x^2 + \frac{1}{x^2} - 2 = 36$

$\Rightarrow x^2 + \frac{1}{x^2} = 38$

Again taking square on both sides

$x^4 + \frac{1}{x^4} + 2 = (38)^2$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 1444$

$\boxed{x^4+\frac{1}{x^4}+2=1444}$

249. (b) $2x + 1 = 0$ $x = \frac{-1}{2}$

$3y - 9 = 0$ $y = +\frac{9}{3} = 3$

$\therefore (x, y) = \left(\frac{-1}{2}, 3\right)$

Point of intersection

250. (c) $x^2 + 9y^2 = 6xy$

$\Rightarrow x^2 + 9y^2 - 6xy = 0$

$\Rightarrow x^2 + (3y)^2 - 2 \times 3y \times x = 0$

$\Rightarrow (x - 3y)^2 = 0$

$x - 3y = 0$

$\Rightarrow x = 3y$

$\therefore x : y$

3 : 1

251. (a) $121a^2 + 64b^2$

$= (11a)^2 + (8b)^2 + 2 \times 11a \times 8b$

$= (11a + 8b)^2$

$\therefore$ So term added to make perfect square = 176 ab

252. (b) $a = 2 + \sqrt{3}$

$\Rightarrow a^2 = \left(2+\sqrt{3}\right)^2$

$\Rightarrow 4 + 3 + 4\sqrt{3}$

$\Rightarrow 7 + 4\sqrt{3}$

$\Rightarrow \frac{1}{a^2} = \frac{1}{7+4\sqrt{3}}$

$= \frac{7-4\sqrt{3}}{(7+4\sqrt{3})(7-4\sqrt{3})} = \frac{7-4\sqrt{3}}{1}$

$\therefore a^2 + \frac{1}{a^2} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$

= 14

253. (a) $p + \frac{1}{4} + \sqrt{p} + k^2$

$\Rightarrow p + \sqrt{p} + \left(k^2+\frac{1}{4}\right)$

$\Rightarrow \left(\sqrt{p}\right)^2 + 2\times\frac{1}{2}\times\sqrt{p} + \left(k^2+\frac{1}{4}\right)$

$\Rightarrow A^2+2\times A\times B+B^2$

$\Rightarrow A = \sqrt{p}$ $B^2 = \left(k^2+\frac{1}{4}\right)$

$B = \frac{1}{2}$

$\therefore k^2 + \frac{1}{4} = \left(\frac{1}{2}\right)^2$

$k^2 + \frac{1}{4} = \frac{1}{4}$

$k^2 = 0$

$k = 0$

254. (b) $\frac{b-c}{a} + \frac{a+c}{b} + \frac{a-b}{c} = 1$

$a - b + c \neq 0$

Let b = c

$\therefore \frac{b-b}{a} + \frac{a+b}{b} + \frac{a-b}{b} = 1$

$\Rightarrow 0 + \frac{a}{b} + 1 + \frac{a}{b} - 1 = 1$

$\Rightarrow \frac{a}{b} + \frac{a}{b} = 1$

$\Rightarrow \frac{1}{b} + \frac{1}{b} = \frac{1}{a}$

we take b = c

$\therefore \boxed{\frac{1}{b}+\frac{1}{c}=\frac{1}{a}}$

255. (a) Reciprocal of $\left(x+\frac{1}{x}\right)$

$= \frac{1}{\left(x+\frac{1}{x}\right)} = \frac{x}{x^2+1}$

256. (a) For minimum value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

$a = b = c$

$a + b + c = 1$ (given)

$\therefore \quad a = b = c = \frac{1}{3}$

$\frac{1}{a} = \frac{1}{b} = \frac{1}{c} = 3$

$\therefore$ Minimum value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

$= 3 + 3 + 3 = 9$

257. (b) $a\left(2+\sqrt{3}\right) = b\left(2-\sqrt{3}\right) = 1$

$a = \frac{1}{2+\sqrt{3}} \qquad b = \frac{1}{2-\sqrt{3}}$

$\Rightarrow \quad a = \frac{1}{b}$

$\Rightarrow \quad \frac{1}{a^2+1} + \frac{1}{b^2+1}$

$\Rightarrow \quad \frac{1}{\frac{1}{b^2}+1} + \frac{1}{b^2+1}$

$\Rightarrow \quad \frac{1}{\frac{1+b^2}{b^2}} + \frac{1}{b^2+1}$

$\Rightarrow \quad \frac{b^2}{b^2+1} + \frac{1}{b^2+1}$

$\Rightarrow \quad \frac{b^2+1}{b^2+1} = 1$

258. (d) $\left(2+\sqrt{3}\right)a = \left(2-\sqrt{3}\right)b = 1$

$\Rightarrow \quad \frac{1}{a} = \left(2+\sqrt{3}\right)$

by rationals

$\Rightarrow \quad \frac{1}{b} = \left(2-\sqrt{3}\right)$

$\Rightarrow \quad \frac{1}{a}+\frac{1}{b} = 2-\sqrt{3}+2+\sqrt{3} = 4$

259. (a) $a+\frac{1}{b} = b+\frac{1}{c} = c+\frac{1}{a}$

To save your time assume values of a, b, c according to equation.

Let $a = 2$, $b = -1$ & $c = \frac{1}{2}$

$2+\frac{1}{-1} = -1+\frac{1}{1/2} = \frac{1}{2}+\frac{1}{2}$

$= 1 = 1 = 1$

$\therefore \quad abc = 2 \times -1 \times \frac{1}{2} = -1$

260. (c) $\frac{x}{y} = \frac{4}{5}$

$\Rightarrow \quad \frac{4}{7}+\left(\frac{2y-x}{2y+x}\right) = \frac{4}{7}+\frac{y\left(2-\frac{x}{y}\right)}{y\left(2+\frac{x}{y}\right)}$

$\Rightarrow \quad \frac{4}{7}+\frac{\left(2-\frac{4}{5}\right)}{2+\frac{4}{5}}$

$\Rightarrow \quad \frac{4}{7}+\left(\frac{10-4}{10+4}\right)$

$\Rightarrow \quad \frac{4}{7}+\frac{6}{14} = \frac{8+6}{14} = \frac{14}{14} = 1$

261. (d) $(x - 2)$ is a factor of $x^2+3Qx-2Q$

for $(x - 2) = 0$

$x = 2$

$x^2 + 3Qx - 2Q = 0$

$\Rightarrow \quad 4 + 3 \times Q \times 2 - 2 \times Q = 0$

$\Rightarrow \quad 4 + 6Q - 2Q = 0$

$4Q = -4$

$Q = -1$

262. (d) $a + b = 12$ .....(I)

$ab = 22$ .....(II)

Squaring both sides of equation (I)

$\Rightarrow \quad a^2 + b^2 + 2ab = 144$

$a^2 + b^2 + 2 \times 22 = 144$

$a^2 + b^2 = 144 - 44 = 100$

263. (b) $x = \sqrt{3}-\frac{1}{\sqrt{3}}$ & $y = \sqrt{3}+\frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{x^2}{y}+\frac{y^2}{x} = \frac{x^3+y^3}{xy}$

$= \frac{(x+y)(x^2-xy+y^2)}{xy}$

$\therefore \quad x+y = \sqrt{3}-\frac{1}{\sqrt{3}}+\sqrt{3}+\frac{1}{\sqrt{3}} = 2\sqrt{3}$

$\therefore \quad xy = \left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$

$= 3 - \frac{1}{3} = \frac{8}{3}$

$\Rightarrow \quad \frac{(x+y)(x^2+y^2+2xy-2xy-xy)}{xy}$

$\Rightarrow \quad \frac{(x+y)\left((x+y)^2-3xy\right)}{xy}$

$\Rightarrow \quad \frac{2\sqrt{3}\left(\left(2\sqrt{3}\right)^2-3\times\frac{8}{3}\right)}{\frac{8}{3}}$

$\Rightarrow \quad \frac{2\sqrt{3}(12-8)}{\frac{8}{3}} \Rightarrow \frac{2\times3\sqrt{3}(4)}{8} = 3\sqrt{3}$

264. (b) $x^2 + ax + b$

$\Rightarrow \quad x^2 + 2 \times \frac{1}{2}a \times x + \left(\sqrt{b}\right)^2$

$\Rightarrow \quad A^2+2\times A\times B+B^2 = (A+B^2)$

$\therefore \quad A = x,\ B = \frac{1}{2}a,\ B^2 = \left(\sqrt{b}\right)^2,$

$B = \sqrt{b}$

$\Rightarrow \quad \left(x+\frac{1}{2}a\right)^2$ be perfect square

at $\sqrt{b} = \frac{1}{2}a \quad b = \frac{1}{4}a^2 \quad \therefore \quad a^2 = 4b$

265. (a) $a + b + c + d = 4$

$\frac{1}{(1-a)(1-b)(1-c)}+\frac{1}{(1-b)(1-c)(1-d)}$

$+\frac{1}{(1-c)(1-d)(1-a)}+\frac{1}{(1-d)(1-a)(1-b)}$

Put $a = 0$, $b = 0$ and $c = 2$ and $d = 2$

$a + b + c + d = 0 + 0 + 2 + 2$

$= 4 = 4$ (satisify)

$\frac{1}{(1-0)(1-0)(1-2)}+\frac{1}{(1-0)(1-2)(1-2)}$

$+\frac{1}{(1-2)(1-2)(1-0)}+\frac{1}{(1-2)(1-0)(1-0)}$

$\Rightarrow \quad \frac{1}{-1}+\left(\frac{1}{+1}\right)+\frac{1}{-1\times-1}+\frac{1}{-1}$

$\Rightarrow \quad -1 + 1 + 1 - 1 = 0$

266. (b) $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}} = 0$

$\Rightarrow \quad a^{\frac{1}{3}}+b^{\frac{1}{3}} = -c^{\frac{1}{3}}$

Take cube on both sides

$\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right)^3 = \left(-c^{\frac{1}{3}}\right)^3$

$\Rightarrow \quad a+b+3a^{\frac{1}{3}}b^{\frac{1}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right) = -c$

$\Rightarrow \quad a+b+3a^{\frac{1}{3}}b^{\frac{1}{3}}\times\left(-c^{\frac{+1}{3}}\right) = -c$

$\Rightarrow \quad a + b + c = 3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$

again taking cube

$(a+b+c)^3 = 27abc$

267. (b) $x - \frac{1}{x} = 1$

$\Rightarrow \frac{x^4 - \frac{1}{x^2}}{3x^2 + 5x - 3}$

Divide and multiply by x

$\Rightarrow \frac{\frac{x^4}{x} - \frac{1}{x^3}}{\frac{3x^2}{x} + \frac{5x}{x} - \frac{3}{x}}$

$= \frac{x^3 - \frac{1}{x^3}}{3x - \frac{3}{x} + 5} \Rightarrow \frac{x^3 - \frac{1}{x^3}}{3\left(x - \frac{1}{x}\right) + 5}$

$\Rightarrow x - \frac{1}{x} = 1$

Take cube on both sides

$\left(x - \frac{1}{x}\right)^3 = (1)^3$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 1$

$x^3 - \frac{1}{x^3} - 3 = 1$

$\Rightarrow x^3 - \frac{1}{x^3} = 4$

$= \frac{x^3 - \frac{1}{x^3}}{3\left(x - \frac{1}{x}\right) + 5} = \frac{4}{3 \times 1 + 5} = \frac{4}{8} = \frac{1}{2}$

268. (d) $x + y = 15$

$\Rightarrow x - 10 = 5 - y$

$x - 10 = -(y - 5)$

Take cube on both sides

$\Rightarrow (x - 10)^3 = (-(y - 5))^3$

$\Rightarrow (x - 10)^3 + (y - 5)^3 = 0$

269. (b) $x^2 + \frac{1}{x^2} = 66$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 66 - 2$

$\left(x - \frac{1}{x}\right)^2 = 64$

$\left(x - \frac{1}{x}\right)^2 = (8)^2$

$\Rightarrow x - \frac{1}{x} = \pm 8$

$\Rightarrow \frac{x^2 - 1 + 2x}{x} = \frac{\frac{x^2}{x} - \frac{1}{x} + \frac{2x}{x}}{\frac{x}{x}}$

$\Rightarrow \frac{\left(x - \frac{1}{x}\right) + 2}{1}$

When $x - \frac{1}{x} = +8$

Then $\left(x - \frac{1}{x}\right) + 2 = 8 + 2 = 10$

When $x - \frac{1}{x} = -8$

$-8 + 2 = -6$

$\therefore$ (10, – 6)

270. (c) $a^2 + a + 1 = 0$

$\left[\begin{array}{l} a^3 + 1^3 = (a+1)(a^2 - a + 1) \\ a^3 - 1 = (a-1)(a^2 + 1 + a) \end{array}\right]$

$\therefore (a^3 - 1) = (a - 1) \times 0$

$a^3 - 1 = 0$

$a^3 = 1$

$(a^3)^3 = 1^3$

$a^9 = 1$

271. (a) $x + \frac{2}{x} = 1$

$x^2 + 2 = x$

$x^2 - x = -2$

$x - x^2 = 2$

$\therefore \frac{x^2 + x + 2}{x^2(1 - x)}$ = divide & mulitply by x

$\Rightarrow \frac{\frac{x^2}{x} + \frac{x}{x} + \frac{2}{x}}{\frac{x^2}{x}(1 - x)} = \frac{x + \frac{2}{x} + 1}{x(1 - x)}$

$\Rightarrow \frac{x + \frac{2}{x} + 1}{x - x^2} = \frac{1 + 1}{2} = 1$

272. (b) $y = 1 - 3k$ and $x = -2k$ (given)

$\therefore$ for $x = y$

$-2k = 1 - 3k$

$k = 1$

273. (b) $\sqrt{(x^2 + y^2 + z)(x + y - 3z)} + \sqrt[3]{xy^3z^2}$

$x = 1$ $y = -3$ $z = -1$ (given)

$\Rightarrow \sqrt{(1 + 9 - 1)(1 - 3 + 3)} + \sqrt[3]{1 \times (-3)^3 \times 1}$

$= 3 + (-3) = 0$

274. (a) $x + \frac{1}{x} = 5$

$\therefore$ Take cube on both sides

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (5)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 5 = 125$

$\Rightarrow x^3 + \frac{1}{x^3} = 110$

$\therefore$ Squaring both sides

$\left(x^3 + \frac{1}{x^3}\right)^2 = (110)^2$

$\Rightarrow x^6 + \frac{1}{x^6} + 2 = 12100$

$x^6 + \frac{1}{x^6} = 12100 - 2 = 12098$

275. (c) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

$x + \frac{1}{x} = 3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$

$x^3 + \frac{1}{x^3} = 18$

$\Rightarrow \frac{x^6 + x^4 + x^2 + 1}{x^3}$

$\Rightarrow \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$

$\Rightarrow x^3 + \frac{1}{x^3} + \frac{1}{x} + x$

$\Rightarrow 18 + 3 = 21$

276. (c) $\frac{p}{a} + \frac{q}{b} + \frac{r}{c} = 1$

$\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 0$

$\Rightarrow \frac{p}{a} = x, \frac{q}{b} = y, \frac{r}{c} = z$

$\Rightarrow (x + y + z) = 1$

Squaring both sides

$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 1$

and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$\frac{xy + yz + zx}{xyz} = 0$

$xy + yz + zx = 0$

$\therefore x^2 + y^2 + z^2 = 1$

So, $\left(\frac{p}{a}\right)^2 + \left(\frac{q}{b}\right)^2 + \left(\frac{r}{c}\right)^2 = 1$

277. (b) $\frac{(x+1)^3 - (x-1)^3}{(x+1)^2 - (x-1)^2} = 2$

$\Rightarrow A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$A^2 - B^2 = (A - B)(A + B)$

$\Rightarrow \frac{(x + 1 - x + 1)\left((x+1)^2 + (x-1)(x+1) + (x-1)^2\right)}{(x + 1 - x + 1)(x + 1 + x - 1)}$

$= 2$

$\Rightarrow \frac{(x^2 + 1 + 2x + x^2 - 1 + x^2 + 1 - 2x)}{(2x)} = 2$

$\Rightarrow \frac{3x^2 + 1}{2x} = 2$

$\Rightarrow \quad 3x^2 + 1 = 4x$

$3x^2 - 4x + 1 = 0$

$3x^2 - 3x - x + 1 = 0$

$3x(x-1) - 1(x-1) = 0$

$(3x-1)(x-1) = 0$

$\Rightarrow \quad 3x - 1 = 0$

$x = \frac{1}{3}$

$\Rightarrow \quad x - 1 = 0$

for $x = 1 = \frac{1}{1}$

By adding numinator and denominator

1 +1 = 2

No option is satisfied

$\therefore \quad x = \frac{1}{3}$

1 + 3 = 4

278. (c) $x = \sqrt{5} + 2$

rationalization of denominator.

$\Rightarrow \quad \frac{1}{x} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$

$\Rightarrow \quad \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

$\Rightarrow \quad x - \frac{1}{x} = \sqrt{5} + 2 - \sqrt{5} + 2 = 4$

$\therefore \quad \frac{2x^2 - 3x - 2}{3x^2 - 4x - 3}$

Multiply by $\frac{1}{x}$ in numerator and denominator

$= \frac{\frac{2x^2}{x} - \frac{2}{x} - \frac{3x}{x}}{\frac{3x^2}{x} - \frac{3}{x} - \frac{4x}{x}} \Rightarrow \frac{2x - \frac{2}{x} - 3}{3x - \frac{3}{x} - 4}$

$= \frac{2\left(x - \frac{1}{x}\right) - 3}{3\left(x - \frac{1}{x}\right) - 4} \Rightarrow \frac{2 \times 4 - 3}{3 \times 4 - 4}$

$= \frac{8-3}{12-4} = \frac{5}{8} = 0.625$

279. (b) a = 2.234

b = 3.121

c = –5.355

$\therefore$ a + b + c= 0

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

280. (d) $x^2 + y^2 + 1 = 2x$

$x^2 - 2x + 1 + y^2 = 0$

$(x-1)^2 + y^2 = 0$

If $A^2 + B^2 = 0$

[As powers are even it can possible only when A = 0 & B = 0]

$\therefore \quad x - 1 = 0$

$x = 1$

$y = 0$

$\therefore \quad x^3 + y^5 = 1 + 0 = 1$

281. (a) $3(a^2 + b^2 + c^2) = (a + b + c)^2$

by options a = b = c

$3(a^2 + a^2 + a^2) = 9a^2$

$\Rightarrow 9a^2 = 9a^2$

282. (a) $x(x-3) = -1$

$\Rightarrow \quad (x - 3) = \frac{-1}{x}$

Taking cube on both sides

$\Rightarrow \quad (x-3)^3 = \left(\frac{-1}{x}\right)^3$

$\Rightarrow \quad x^3 - 27 - 9x(x-3) = \frac{-1}{x^3}$

$\Rightarrow \quad x^3 - 27 - 9 \times -1 = \frac{-1}{x^3}$

$\Rightarrow \quad x^3 - 27 + 9 = \frac{-1}{x^3}$

$\Rightarrow \quad x^3 - 18 = \frac{-1}{x^3}$

$\Rightarrow \quad x^3(x^3 - 18) = -1$

283. (a) $a^2 + 4b^2 + 4b - 4ab - 2a - 8$

$= a^2 - 4ab + 4b^2 - 2a + 4b - 8$

$= (a - 2b)^2 - 2(a - 2b) - 8$

Put t = a – 2b

$= t^2 - 2t - 8$

$= t^2 - 4t + 2t - 8$

$= t(t-4) + 2(t-4)$

$= (t+2)(t-4)$

$= (a - 2b - 4)(a - 2b + 2)$

(Put the value of assume t)

284. (d) $\frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2} + \frac{2ax}{a^4 + a^2x^2 + x^4}$

$= \frac{a^2 - ax + x^2 - a^2 - ax - x^2}{(a^2 + x^2 + ax)(a^2 + x^2 - ax)} + \frac{2ax}{a^4 + a^2x^2 + x^4}$

$= \frac{-2ax}{(a^2 + x^2)^2 - (ax)^2} + \frac{2ax}{a^4 + x^4 + a^2x^2}$

$= \frac{-2ax}{a^4 + x^4 + 2x^2a^2 - a^2x^2} + \frac{2ax}{a^4 + x^4 + a^2x^2}$

$= \frac{-2ax}{a^4 + x^4 + x^2a^2} + \frac{2ax}{a^4 + x^4 + a^2x^2} = 0$

285. (b) $x = 11$

$x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$

$= x^5 - 11x^4 - x^4 + 11x^3 + x^3 - 11x^2 - x^2 + 11x + x - 1$

$= (11)^5 - 11 \times (11)^4 - (11)^4 + 11 \times (11)^3 + 11^3 - 11 \times (11)^2 - (11 \times 11) + (11 \times 11) + (11) - 1$

$= 0 - 0 + 0 + 0 + 11 - 1 = 10$

286. (c) From option (c) LHS $(x + 2)^2 = x^2 + 4x + 4$

RHS $= x^2 + 2x + 4$

$\therefore$ LHS $\neq$ RHS

287. (d) $x^2 + y^2 + z^2 - xy - xz - yz = \frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$

$= \frac{1}{2}[(997 - 998)^2 + (998 - 999)^2 + (999 - 997)^2]$

$= \frac{1}{2}(1+1+4) = 3$

288. (c) $x + \frac{1}{x} = 3$

$\frac{3x^2 + 3 - 4x}{x^2 + 1 - x} = \frac{\frac{3x^2}{x} + \frac{3}{x} - 4}{\frac{x^2}{x} + \frac{1}{x} - \frac{x}{x}} = \frac{3\left(x + \frac{1}{x}\right) - 4}{\left(x + \frac{1}{x}\right) - 1}$

$= \frac{3 \times 3 - 4}{3 - 1} = \frac{9 - 4}{3 - 1} = \frac{5}{2}$

289. (d) $x = 3 + 2\sqrt{2}$

$\therefore \quad \frac{1}{x} = 3 - 2\sqrt{2}$

$x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$

$x + \frac{1}{x} = 6$

$\left(x + \frac{1}{x}\right)^3 = (6)^3$

$x^3 + \frac{1}{x^3} + 3 \times 6 = 216$

$x^3 + \frac{1}{x^3} = 216 - 18 = 198$

$\therefore \quad \frac{x^6 + x^4 + x^2 + 1}{x^3}$

$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$= x^3 + \frac{1}{x^3} + x + \frac{1}{x}$

$= 198 + 6 = 204$

290. (c) $x = p + \frac{1}{p}$, $y = p - \frac{1}{p}$

$\therefore \quad x + y = p + \frac{1}{p} + p - \frac{1}{p} = 2p$

$x - y = p + \frac{1}{p} - p + \frac{1}{p} = \frac{2}{p}$

$\Rightarrow \quad x^4 + y^4 - 2x^2y^2$

$\Rightarrow \quad (x^2 - y^2)^2 = [(x + y)(x - y)]^2$

$\Rightarrow \quad \left(2p \times \frac{2}{p}\right)^2 = (4)^2 = 16$

291. (c) $(a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2 = ?$

$\Rightarrow \quad a + b + c = 0$ (given)

$\Rightarrow \quad a + b = -c$

$\Rightarrow \quad b + c = -a$

$\Rightarrow \quad a + c = -b$

$\Rightarrow \quad (a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2$

$\Rightarrow (-c-c)^2 + (-a-a)^2 + (-b-b)^2$

$\Rightarrow (-2c)^2 + (-2a)^2 + (-2b)^2$

$\Rightarrow 4c^2 + 4a^2 + 4b^2$

$\Rightarrow 4(a^2 + b^2 + c^2)$

292. (b) $p^3 + 3p^2 + 3p = 7$

$p^3 + 3p^2 + 3p + 1 = 7 + 1$

$= (p + 1)^3 = (2)^3$

$= p + 1 = 2$

$\Rightarrow p = 1$

$\therefore p^2 + 2p = 1 + 2 = 3$

293. (a) $x = 2015$

$y = 2014$

$z = 2013$

$= x^2 + y^2 + z^2 - xy - yz - zx =$

$\frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}[(2015 - 2014)^2 + (2014 - 2013)^2 + (2013 - 2015)^2]$

$= \frac{1}{2}(1 + 1 + 4) = 3$

294. (a) $3a^2 = b^2$ (given)

$$\frac{(a+b)^3 - (a-b)^3}{(a+b)^2 + (a-b)^2}$$

$$\Rightarrow \frac{a^3 + b^3 + 3ab(a+b) - (a^3 - b^3 - 3ab(a-b))}{a^2 + b^2 + 2ab + a^2 + b^2 - 2ab}$$

$$\Rightarrow \frac{2b^3 + 6a^2b}{2a^2 + 2b^2} \Rightarrow \frac{b^3 + 3a^2b}{a^2 + b^2}$$

$$\Rightarrow \frac{b^3 + b^3}{\frac{b^2}{3} + b^2} = \frac{2b^3}{b^2\left(\frac{1}{3} + 1\right)}$$

$$\Rightarrow \frac{2b}{\frac{4}{3}} = \frac{3b}{2}$$

295. (a) $x + \frac{1}{x} = 2\frac{1}{12} = \frac{25}{12}$

$$x^2 + \frac{1}{x^2} + 2 = \frac{675}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{625}{144} - 2$$

$$x^2 + \frac{1}{x^2} = \frac{625 - 288}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{337}{144}$$

$$x^2 + \frac{1}{x^2} - 2 = \frac{337}{144} - 2$$

$$\left(x - \frac{1}{x}\right)^2 = \frac{337 - 288}{144} = \frac{49}{144}$$

$$x - \frac{1}{x} = \frac{7}{12}$$

$$\therefore \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = \frac{25}{12} \times \frac{7}{12} = \frac{175}{144}$$

$$\therefore \left(x^2 - \frac{1}{x^2}\right) = \frac{175}{144}$$

$$\therefore x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$$

$$= \frac{175}{144} \times \frac{337}{144} = \frac{58975}{20736}$$

296. (c) $x = 9999$

$$\frac{4x^3 - x}{(2x+1)(6x-3)} = \frac{x(4x^2 - 1)}{3(2x+1)(2x-1)}$$

$$= \frac{x(4x^2 - 1)}{3(4x^2 - 1)} = \frac{x}{3}$$

$$\therefore \frac{9999}{3} = 3333$$

297. (b) $a^3 + b^3 = 9$

$a + b = 3$

Assume values, $a = 2$, $b = 1$

$\therefore (2)^3 + 1 = 9$

$2 + 1 = 3$

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2} + 1 = \frac{3}{2}$$

298. (c) $t^2 - 4t + 1 = 0$

$t^2 + 1 = 4t$

$$\frac{t^2 + 1}{t} = \frac{4t}{t}$$

$$t + \frac{1}{t} = 4$$

[take cube both sides]

$$t^3 + \frac{1}{t^3} + 3t \times \frac{1}{t}\left(t + \frac{1}{t}\right) = 64$$

$$t^3 + \frac{1}{t^3} = 64 - 12 = 52$$

$$t^3 + \frac{1}{t^3} = 52$$

299. (d) $\sqrt[3]{a} + \sqrt[3]{b} = \sqrt[3]{c}$

Take cube both sides

$$\left(\sqrt[3]{a} + \sqrt[3]{b}\right)^3 = \left(\sqrt[3]{c}\right)^3$$

$$\Rightarrow a + b + 3a^{\frac{1}{3}}b^{\frac{1}{3}}\left(\sqrt[3]{a} + \sqrt[3]{b}\right) = c$$

$$\Rightarrow a + b + 3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}} = c$$

$$\Rightarrow a + b - c = -3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$$

Again take cube both sides

$\Rightarrow (a + b - c)^3 = -27\,abc$

$\Rightarrow (a + b - c)^3 + 27abc = 0$

Alternate:

Put value of $a = 0$

$b = 1$, $c = 1$

value of $(a + b - c)^3 + 27\,abc$

$= (0 + 1 - 1)^3 + 27 \times 0 \times 1 \times 1 = 0$

300. (b) $4x + 5y = 83$

$3x : 2y = 21 : 22$

$x : y = 7 : 11$

let $x = 7$ and $y = 11$

$4 \times 7 + 5 \times 11 = 83$

So, value satisfied

$y - x = 11 - 7 = 4$

301. (c) $x = \sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^2 + b^3}}$

Take cube on both sides

$$x^3 = (a + \sqrt{a^2 + b^3}) + (a - \sqrt{a^2 + b^3}) + 3\left(a + \sqrt{a^2 + b^3}\right)^{\frac{1}{3}} \cdot \left(a - \sqrt{a^2 + b^3}\right)^{\frac{1}{3}} \cdot \left(\sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^3 + b^3}}\right)$$

$$x^3 = 2a + 3\left(a^2 - \left(a^2 + b^3\right)\right)^{\frac{1}{3}} \times x$$

$$x^3 = 2a + 3\left(-b^3\right)^{\frac{1}{3}}(x)$$

$$x^3 = 2a + 3\left(-b^3\right)^{\frac{1}{3}}(x)$$

$x^3 = 2a - 3bx$

$x^3 + 3bx = 2a$

302. (d) Given

$$\frac{x^{24} + 1}{x^{12}} = 7$$

$$\frac{x^{24} + 1}{x^{12}} \Rightarrow \frac{x^{24}}{x^{12}} + \frac{1}{x^{12}}$$

$$\Rightarrow x^{12} + \frac{1}{x^{12}} = 7$$

$\Rightarrow$ Cubing both sides

$$\Rightarrow \left(x^{12} + \frac{1}{x^{12}}\right)^3 = 7^3$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} + \frac{3 \times x^{12} \times 1}{x^{12}}\left(x^{12} + \frac{1}{x^{12}}\right) = 343$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} + 3 \times 7 = 343$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} = 343 - 21$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} = \frac{x^{72} + 1}{x^{36}} = \mathbf{322}$$

303. (b) Given, $x = 2$

Find $x^3 + 27x^2 + 243x + 631$

$\Rightarrow$ to put value $x = 2$

$\Rightarrow 2^3 + 27 \times 2^2 + (243 \times 2) + 631$

$\Rightarrow 8 + 108 + 486 + 631$

$\Rightarrow$ **1233**

304. (c) Given

$5x + 9y = 5$ ....(i)

$125x^3 + 729y^3 = 120$ ...(ii)

From equation (i) cubing both sides

$\Rightarrow (5x + 9y)^3 = 5^3$

$\Rightarrow 125x^3 + 729y^3 + 3 \times 5x \times 9y(5x + 9y) = 125$

$\Rightarrow 125x^3 + 729y^3 + 135xy \times 5 = 125$

$\Rightarrow$ 120 + 135xy × 5 = 125

$\Rightarrow$ 135xy × 5 = 5

$\Rightarrow$ $xy = \frac{1}{135}$

$\Rightarrow$ therefore product of x & y = $\frac{1}{135}$

305. (b) Given, $x^2+y^2+z^2 = 2(x+z-1)$

Find- $x^3+y^3+z^3 = ?$

$\Rightarrow x^2+y^2+z^2 = 2(x+z-1)$

$\Rightarrow x^2+y^2+z^2 = 2x+2z-2$

$\Rightarrow x^2+y^2+z^2 = 2x+2z-1-1$

$\Rightarrow (x^2+1-2x) + y^2 + (z^2+1-2z) = 0$

$\Rightarrow (x-1)^2 + y^2 + (z-1)^2 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1$

$\Rightarrow y^2 = 0$

$\Rightarrow y = 0$

$\Rightarrow (z-1)^2 = 0$

$\Rightarrow z = 1$

Value substituted in question,

$\Rightarrow x^3+y^3+z^3$

$\Rightarrow 1^3 + 0 + 1^3$

$\Rightarrow 2$

306. (b) Given, $x+\frac{1}{x}=1$

Find: $\frac{2}{x^2-x+2}=?$

$x+\frac{1}{x}=1$

$x^2+1 = x$

$(x^2-x) = -1$

Putting value in,

$\frac{2}{(x^2-x)+2} = 2$

307. (d) Given,

$x=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}, \quad y=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Find: $\frac{x^2+y^2+xy}{x^2+y^2-xy}=?$

$\Rightarrow \frac{x^2+y^2+2xy-xy}{x^2+y^2-2xy+xy}$

$\Rightarrow \frac{(x+y)^2-xy}{(x-y)^2+xy}=?$

Now, $x+y=\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}+\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$\Rightarrow x+y=\frac{(\sqrt{5}-\sqrt{3})^2+(\sqrt{5}+\sqrt{3})^2}{\sqrt{5}^2-\sqrt{3}^2}$

$\Rightarrow x+y=\frac{2(\sqrt{5}^2+\sqrt{3}^2)}{2}$

$\Rightarrow x + y = 8$ ....(i)

Again, $x-y=\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}-\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$=\frac{4\times\sqrt{5}\times\sqrt{3}}{2}$

$\Rightarrow (x-y)=2\sqrt{15}$ ...(ii)

And, $xy=\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}\times\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$xy = 1$

Substitutes values in the question.

$\Rightarrow \frac{(x+y)^2-xy}{(x-y)^2+xy}$

$\Rightarrow \frac{8^2-1}{(2\sqrt{15})^2+1} \Rightarrow \frac{63}{61}$

308. (c) Given, $4a-\frac{4}{a}+3=0$

Find: $a^3-\frac{1}{a^3}+3=?$

$\Rightarrow 4a-\frac{4}{a}=-3$

$\Rightarrow a-\frac{1}{a}=\frac{-3}{4}$

$\Rightarrow \left(a-\frac{1}{a}\right)^3 = \left(\frac{-3}{4}\right)^3$

[Cubing both sides]

$\Rightarrow a^3-\frac{1}{a^3}-3a\times\frac{1}{a}\left(a-\frac{1}{a}\right)=\frac{-27}{64}$

$\Rightarrow a^3-\frac{1}{a^3}-3\times\left(-\frac{3}{4}\right)=\frac{-27}{64}$

$\Rightarrow a^3-\frac{1}{a^3}=\frac{-27}{64}-\frac{9}{4}$

$\Rightarrow a^3-\frac{1}{a^3}+3=\frac{-27}{64}-\frac{9}{4}+3$

$\Rightarrow \frac{192-171}{64} \Rightarrow a^3-\frac{1}{a^3}+3=\frac{21}{64}$

309. (a) $\left(\left[1+\frac{1}{10+\frac{1}{10}}\right]\left[1+\frac{1}{10+\frac{1}{10}}\right]-\left[1-\frac{1}{10+\frac{1}{10}}\right]\left[1-\frac{1}{10+\frac{1}{10}}\right]\right)$

$\div\left(\left[1+\frac{1}{10+\frac{1}{10}}\right]+\left[1-\frac{1}{10+\frac{1}{10}}\right]\right)$

Let $\left[1+\frac{1}{10+\frac{1}{10}}\right]=a, \left[1-\frac{1}{10+\frac{1}{10}}\right]=b$

$\Rightarrow (a^2-b^2) \div (a+b) = a - b = ?$

$\Rightarrow a=1+\frac{10}{101} \Rightarrow \frac{111}{101}$

$\Rightarrow b=1-\frac{10}{101} \Rightarrow \frac{91}{101}$

a - b = $\frac{111}{101}-\frac{91}{101}=\frac{20}{101} \Rightarrow \frac{20}{101}$

310. (b) According to the question,

$\because$ x = z = 225

y = 226

$\Rightarrow x^3+y^3+z^3-3xyz = ?$

As we know,

$x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)$

$[(x-y)^2+(y-z)^2+(z-x)^2]$

$=\frac{1}{2}[225+225+226][(225-226)^2 +$

$(+226-225)^2+(225-225)^2]$

$=\frac{676}{2}\times[1+1+0] = 676$

311. (b) According to the question,

$\because a+b-c = 14$

Find: $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4-b^4-c^4 = ?$

$\Rightarrow a+b-c = 14$

Let a = 7, b = 7, c = 0

$\therefore 2\times7^2\times0+2\times0^2\times7^2+2\times7^2\times7^2-7^4-7^4-0^4 = 0$

312. (b) According to the question,

$\Rightarrow \left[(x+3)^3+\frac{1}{(x+3)^3}\right]$

Let, t = x + 3, x = t – 3

$x^2 + x = 5$ (given)

$(x + 1)x = 5$

$(t-3+1).(t-3)=5$

$(t-2).(t-3) = 5$

$t^2 - 3t - 2t + 6 = 5$

$t^2-5t = -1$

$t + \frac{1}{t} = 5$

$(x+3)^3+\frac{1}{(x+3)^3}$

$= t^3 + \frac{1}{t^3}$

$=\left(t+\frac{1}{t}\right)^3-3\times t\times\frac{1}{t}\left(t+\frac{1}{t}\right)$

$= 5^3 - 3 \times 5 = 125 - 15$

$= 110 = (x+3)^3+\frac{1}{(x+3)^3} = 110$

313. (d) Given, m = – 4, n = – 2

Find $m^3-3m^2+3m+3n+3n^2+n^3$

Putting value of m and n

$\Rightarrow (-4)^3 - 3(-4)^2 + 3(-4) + 3\times(-2) + 3(-2)^2 + (-2)^3$

$\Rightarrow -64 - 48 - 12 - 6 + 12 - 8$

$\Rightarrow -64 - 60 - 2$

$\Rightarrow -126$

314. (b) $2x - ky + 7 = 0$ ....(i)

$6x - 12y + 15 = 0$ ....(ii)

There has no solution for

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{6} = \frac{-k}{-12}$

$\frac{1}{3} = \frac{k}{12} \Rightarrow K = 4$

315. (a) Here, x = 332 , y = 333 , z = 335

Find $x^3 + y^3 + z^3 - 3\,xyz = \frac{1}{2}(x+y+z)$

$\left[(x-y)^2+(y-z)^2+(z-x)\right]$

$=\left(\frac{332+333+335}{2}\right)\left[(333-332)^2+(335-333)^2\right.$

$\left.+(335-332)^2\right]$

$=\frac{1000}{2}(1^2+2^2+3^2)=\frac{1000}{2}(14)$ = 7000

316. (d) If $2+x\sqrt{3}=\frac{1}{2+\sqrt{3}}$

Find x = ?

$\Rightarrow 2+\sqrt{3}\,x=\frac{1}{2+\sqrt{3}}$

$\Rightarrow 2+\sqrt{3}\times x=\frac{2-\sqrt{3}}{1}$

$\Rightarrow 2+x\sqrt{3}=2-\sqrt{3}$

$\Rightarrow x=-1$

317. (b) Given,

$\frac{m-a^2}{b^2+c^2}+\frac{m-b^2}{c^2+a^2}+\frac{m-c^2}{a^2+b^2}=3$

$\Rightarrow$ m = ?

$\Rightarrow \frac{m-a^2}{b^2+c^2}+\frac{m-b^2}{c^2+a^2}+\frac{m-c^2}{a^2+b^2}=1+1+1$

Put m = $a^2+b^2+c^2$ from option (b)

LHS. = $\frac{a^2+b^2+c^2-a^2}{b^2+c^2}+$

$\frac{a^2+b^2+c^2-b^2}{c^2+a^2}++\frac{a^2+b^2+c^2-c^2}{a^2+b^2}$

$\frac{b^2+c^2}{b^2+c^2}+\frac{c^2+a^2}{c^2+a^2}+\frac{a^2+b^2}{a^2+b^2}=1+1+1$

= RHS

$\Rightarrow$ m = $a^2+b^2+c^2$

318. (c) Given m−5n = 2

find $m^3 - 125\,n^3 - 30mn$

$\Rightarrow$ m − 5n = 2

$\Rightarrow (m-5n)^3 = 2^3$ (cubing both sides)

$\Rightarrow m^3-125n^3-3m\times5n\,(m-5n) = 8$

$\Rightarrow m^3 - 125n^3 - 15mn\times2 = 8$

$\Rightarrow m^3 - 125n^3 - 30mn = 8$

319. (b) Given, $x^3 + y^3 = 72$

$xy = 8$

$(x > y)$

If we take x = 4 and y = 2

$4^3 + 2^3 = 72$

(72 = 72)

and xy = 8

$\Rightarrow$ 4 × 2 = 8

so $x - y$ = ?

4 − 2 = 2

320. (d) Given

$x=\sqrt{a\sqrt[3]{b\sqrt{a\sqrt[3]{b}}}}\,.....\infty$ ....(i)

on squaring both side

$\Rightarrow x^2 = a\sqrt[3]{b\sqrt{a\sqrt[3]{b}}}\,....\infty$

on cubing both sides

$\Rightarrow x^6 = a^3b\sqrt{a\sqrt[3]{b\sqrt{a\sqrt[3]{b}}}}\,.....\infty$

$\Rightarrow x^6 = a^3b\,x$ from equation (i)

$\Rightarrow$ On dividing above eq. by x we get

$\Rightarrow \frac{x^6}{x}=\frac{a^3bx}{x}$

$\Rightarrow x^5 = a^3b$

$\Rightarrow x = \sqrt[5]{a^3b}$

321. (d) **Given:** $x+\frac{1}{x}=2$ ....(i)

The value of $x^{12}-\frac{1}{x^{12}}$=?

$\Rightarrow$ if x = 1 $\Rightarrow x+\frac{1}{x}=2$

1 + 1 = 2

Then, $x^{12}-\frac{1}{x^{12}}$

$\Rightarrow 1^{12}-\frac{1}{1^{12}}$

$\Rightarrow$ 1 − 1 = 0

322. (a) Given: $x+\frac{1}{x}=1$ ...(i)

Find $\frac{x^2+3x+1}{x^2+7x+1}$=?

From equation (i)

$\Rightarrow x+\frac{1}{x}=1$

$\Rightarrow x^2 + 1 = x$

$\Rightarrow \frac{(x^2+1)+3x}{(x^2+1)+7x}$

$\Rightarrow \frac{x+3x}{x+7x} \Rightarrow \frac{4x}{8x}=\frac{1}{2}$

323. (c) $x+\frac{1}{x}=2$

Find $x^7+\frac{1}{x^5}=2$

$x+\frac{1}{x}=2$

$\Rightarrow \downarrow \quad \downarrow \quad \Rightarrow$ Let x = 1

1 1

To, put value in question,

$\Rightarrow x^7+\frac{1}{x^5} \Rightarrow 1^7+\frac{1}{1^5}$

$\Rightarrow$ 1+1 = 2

324. (d) Given expression,

$\Rightarrow 4x^2 + 8x$

Let P should be added,

$\Rightarrow 4x^2 + 8x + p$

$\Rightarrow (2x)^2 + 2 \times (2x) \times 2$

$[(a+b)^2 = a^2+b^2+2ab]$

Term that should be added = 2^2 = 4

P = 4

325. (b) 999x + 888y = 1332

888x + 999y = 555

1887 $(x + y)$ = 1887

$x + y$ = 1

326. (b) According to the question,

$a^2+b^2+c^2 = ab+bc+ca$

Put a =1, b =1, c =1

$\therefore 1^2+1^2+1^2 = 1\times1+1\times1+1\times1$

1+1+1 = 1+1+1

3 = 3 [Satisfy]

$\therefore \frac{a+c}{b}=\frac{1+1}{1}=2$

327. (a) According to the question,

$\Rightarrow x=\frac{1}{2+\sqrt{3}},\quad y=\frac{1}{2-\sqrt{3}}$

$\Rightarrow x=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}},$

$y=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\Rightarrow x=2-\sqrt{3},\ y=2+\sqrt{3}$

$8xy\ (x^2+y^2)$

$=8(2-\sqrt{3})(2+\sqrt{3})\left[(2-\sqrt{3})^2+(2+\sqrt{3})^2\right]$

$\Rightarrow 8\times1\left[7-2\sqrt{3}+7+2\sqrt{3}\right]$ = 112

328. (d) According to the question,

$a=\frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$

Put x = 2

$a=\frac{\sqrt{2+2}+\sqrt{2-2}}{\sqrt{2+2}-\sqrt{2-2}}$

$a=\frac{\sqrt{4}}{\sqrt{4}}=1$

$a^2 - ax = 1^2-1\times2 = 1 - 2 = -1$

329. (a) Let a = 0

b = 1

$\Rightarrow a^3+b^3-ab-(a^2-b^2)^2$

$\Rightarrow 0+1-0-(0-1)^2$

$\Rightarrow$ 1 − 1 = 0

330. (b) $a-\frac{1}{(a-3)}=5$

$a-3-\frac{1}{a-3}=5-3$

$(a-3)-\frac{1}{(a-3)}=2$

Cubing both sides

$\left[(a-3)-\frac{1}{(a-3)}\right]^3=2^3$

$$(a-3)^3 - \frac{1}{(a-3)^3} - 3.(a-3)\times\frac{1}{(a-3)}$$

$$\left[(a-3) - \frac{1}{(a-3)}\right] = 8$$

$$(a-3)^3 - \frac{1}{(a-3)^3} - 3\,[\,2\,] = 8$$

$$(a-3)^3 - \frac{1}{(a-3)^3} = 8+6 = 14 \text{ Ans.}$$

331. (d) $\frac{3x-2y}{2x+3y} = \frac{5}{6}$

$18x - 12y = 10x + 15y$

$8x = 27y$

$\frac{x}{y} = \frac{27}{8}$

$$\left[\frac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}}\right]^2$$

$$\left(\frac{\sqrt[3]{27} + \sqrt[3]{8}}{\sqrt[3]{27} - \sqrt[3]{8}}\right)^2$$

$$\Rightarrow \left(\frac{3+2}{3-2}\right)^2 = (5)^2 = 25$$

332. (d) According to the Question

$x = \sqrt{3} + \sqrt{2}$

$y = \sqrt{3} - \sqrt{2}$

$(x^3 - 20\sqrt{2}) - (y^3 + 2\sqrt{2})$

$=[(\sqrt{3}+\sqrt{2})^3 - 20\sqrt{2} - (\sqrt{3} - \sqrt{2})^3 - 2\sqrt{2}]$

$=3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} + 6\sqrt{3} - 20\sqrt{2} - 3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} - 6\sqrt{3} - 2\sqrt{2}$

$= 9\sqrt{3} - 9\sqrt{2} - 9\sqrt{3} + 9\sqrt{2} = 0$

333. (d) **SHORTCUT METHOD**

Always do these types of question with the help of

Put a = b = c = 1

$3(1^2 + 1^2 + 1^2) = (a + b + c)^2$

3 = 3 satified

So this is answer $\rightarrow$ a = b = c

334. (d) According to the question,

$x = \sqrt{a} + \frac{1}{\sqrt{a}}$ & $y = \sqrt{a} - \frac{1}{\sqrt{a}}$

$\therefore x^4 - x^2y^2 - 1 + y^4 - x^2y^2 + 1$

$= x^4 - 2x^2y^2 + y^4$

$= [x^2 - y^2]^2$

$$\left[\left(\sqrt{a} + \frac{1}{\sqrt{a}}\right)^2 - \left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)^2\right]^2$$

$$= \left[a + \frac{1}{a} + 2 - a - \frac{1}{a} + 2\right]^2$$

$= [4]^2 = 16$

335. (d) Let m = $\sqrt{5+\sqrt{5+\sqrt{5}}}$

Factor = (a)×(a+1)

Here m = a + 1

or m − 1 = a ....(i)

Let n = $\sqrt{5-\sqrt{5-\sqrt{5}}}$

Factor = (a)×(a+1)

Here n = a ....(ii)

From (i) & (ii)

m − 1 = n

or m − n − 1 = 0

336. (b) $\frac{3-5x}{2x} + \frac{3-5y}{2y} + \frac{3-5z}{2z} = 0$

or $\frac{3}{2x} - \frac{5}{2} + \frac{3}{2y} - \frac{5}{2} + \frac{3}{2z} - \frac{5}{2} = 0$

or $\frac{3}{2x} + \frac{3}{2y} + \frac{3}{2z} = \frac{3\times5}{2}$

or $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3\times5\times2}{2\times3}$

or $\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = \frac{3\times5\times2\times2}{2\times3}$

=10

337. (a) According to the question

If 2S = a + b + c

$S = \frac{a+b+c}{2}$

Let a = 10, b = 10, c = 10

$\because S = \frac{10+10+10}{2}$

$S = \frac{30}{2} = 15$

$\because$ S(S − C) + (S − a) (S − b)

15(15 − 10) + (15 − 10)

(15 − 10) = 75 + 25 = 100

Now check from option.

Option (a) ab = 10×10 = 100 (Satisfied)

338. (d) p + m = 6 .....(i)

$p^3 + m^3 = 72$

$(p + m)(p^2 + m^2 - pm) = 72$

$(p + m)[(p + m)^2 - 3pm] = 72$

$[\because p^2 + m^2 = (p + m)^2 - 2pm]$

$6[(6)^2 - 3pm] = 72$ from (i)

36 − 3pm = 12

pm = 8

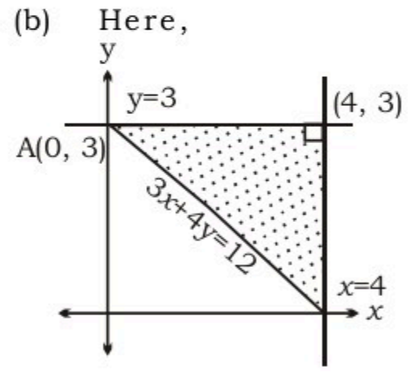
339. (d) $x^m \times x^n = 1$

$x^{m+n} = x^o$ $(\because x^o = 1)$

m + n = 0

m = −n

340. (b) Here,



base = 3unis

height = 4 units

Area of $\Delta = \frac{1}{2} \times b \times h$

$= \frac{1}{2}\times3\times4 = 6$ sq. units

341. (b) Here k = 2k −1

Let r = 1

[x = y
↓ ↓
abscissa = ordinate]

k = 2k − 1

k = 1

342. (a) According to the question.

If x = 5

$\Rightarrow x^2 - 2 + \frac{1}{x^2}$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 \Rightarrow \left(5 - \frac{1}{5}\right)^2$

$\Rightarrow \left(\frac{24}{5}\right)^2 \Rightarrow \frac{576}{25}$

343. (b) According to the question,

$x = 2,\ y = 1,\ z = -3$

$\therefore x^3 + y^3 + z^3 - 3xyz = ?$

As we know that

a + b + c = 0 then $a^3 + b^3 + c^3 - 3abc = 0$

$\therefore 2 + 1 - 3 = 0$

$x^3 + y^3 + 3^3 - 3xyz = 0$

344. (d) According to the question,

$a + \frac{1}{a} = -1$ and $a^2 + a = -1$

$a^3 = 1$

$(-a - a)(-a^2 - a^2)$

$(-2a)(-2a^2)$

$4a^3 = 4 \times 1 = 4$

345. (a) According to the question

$\Rightarrow (x^3 + y^6)(x^3 - y^6)$

$\Rightarrow x^6 + x^3y^6 - x^3y^6 - y^{12}$

$\Rightarrow x^6 - y^{12}$

346. (b) According to the question,

$\Rightarrow \frac{1}{x+y} + \frac{1}{x-y}$

$\Rightarrow \frac{x-y+x+y}{x^2-y^2} \Rightarrow \frac{2x}{x^2-y^2}$

347. (a) Given, $x + y = 2a$ to

Find $\frac{a}{(x-a)} + \frac{a}{(y-a)} = ?$

$\Rightarrow x + y = 2a$
↓ ↓ ↓
3 1 2

$\Rightarrow$ Let x = 3, y = 1, a = 2

$\Rightarrow \frac{a}{(x-a)} + \frac{a}{y-a}$

$\Rightarrow \frac{2}{(3-2)} + \frac{2}{(1-2)}$

$\Rightarrow \frac{2}{1} + \frac{2}{-1} = 0$

348. (d) $p^3 - q^3 = (p - q)\{(p - q)^2 - xpq\}$

$\Rightarrow p^3 - q^3 = (p - q)[p^2 + q^2 - 2pq - xpq]$

$\Rightarrow p^3 - q^3 = (p - q)[p^2 + q^2 - 2pq - (-3)pq]$

$\Rightarrow p^3 - q^3 = (p - q)(p^2 + q^2 + pq)$

$\Rightarrow$ So, $x = -3$

$\Rightarrow$ because $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

349. (a) Given

$\Rightarrow x + y + z = 6$

$\Rightarrow xy + yz + zx = 10$

To find $x^3 + y^3 + z^3 - 3xyz = ?$

$\Rightarrow$ Using formula.

$\Rightarrow (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow 6^2 = x^2 + y^2 + z^2 + 2 \times 10$

$\Rightarrow 36 = x^2 + y^2 + z^2 + 20$

$\Rightarrow x^2 + y^2 + z^2 = 16$

$\Rightarrow x^2 + y^2 + z^2 - 3xyz =$

$(x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$

$= 6[16 - (xy + yz + zx)]$

$= 6[16 - 10] = 6 \times 6 = 36$

350. (d) Given:

$\Rightarrow \frac{x+1}{x-1} = \frac{a}{b}$

$\Rightarrow \frac{x}{1} = \frac{a+b}{a-b}$

(using componendo & dividendo)

$\Rightarrow x = \frac{a+b}{a-b}$ ....(i)

Again,

$\Rightarrow \frac{1-y}{1+y} = \frac{b}{a}$

$\Rightarrow \frac{1+y}{1-y} = \frac{a}{b}$

$\Rightarrow \frac{1}{y} = \frac{a+b}{a-b}$

$\Rightarrow y = \frac{a-b}{a+b}$ ....(i)

$\Rightarrow$ From question, $\frac{x-y}{1+xy}$

$\Rightarrow \frac{\frac{a+b}{a-b} - \frac{(a-b)}{(a+b)}}{1 + \left(\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}\right)}$

$\Rightarrow \frac{(a+b)^2 - (a-b)^2}{(a^2 - b^2)(1+1)}$

$\Rightarrow \frac{4ab}{2(a^2 - b^2)} \Rightarrow \frac{2ab}{a^2 - b^2}$

351. (b) Given

$\Rightarrow a^2 + a + 1 = 0$ ....(i)

$\Rightarrow$ to find $a^5 + a^4 + 1 = ?$

by formula

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$\Rightarrow a^3 - 1 = (a - 1)(a^2 + a + 1)$

$[b = 1]$

$\Rightarrow a^3 - 1 = (a - 1) \times 0$

$\Rightarrow a^3 - 1 = 0$

$\Rightarrow a^3 = 1$

$\Rightarrow a^5 + a^4 + 1$

$\Rightarrow a^3 \times a^2 + a^3 + a + 1$

$\Rightarrow a^2 \times 1 + a \times 1 + 1$

$\Rightarrow a^2 + a + 1 = 0$ from equation..(i)

$\Rightarrow$ Answer will be 0

352. (d) Given

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

to find $a : b : c = ?$

$\Rightarrow$ According to the question,

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow$ Let $a = b = c = 1$

$\Rightarrow 1^2 + 1^2 + 1^2 = 1 \times 1 + (1 \times 1) + (1 \times 1)$

$\Rightarrow 3 = 3$

$\Rightarrow$ So ratio of $a : b : c = 1 : 1 : 1$

353. (d) $x - \frac{1}{x} = 2$ to find $x^3 - \frac{1}{x^3} = ?$

$\Rightarrow x - \frac{1}{x} = 2 \Rightarrow \left(x - \frac{1}{x}\right)^3$

$= 2^3$ cubing both side

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right) = 8$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 1 \times (2) = 8$

$\Rightarrow x^3 - \frac{1}{x^3} = 14$

354. (c) $x = a(b - c)$

$y = b(c - a)$

$z = c(a - b)$

Let $\frac{x}{a} = b - c$ $\quad \frac{x}{a} = A$

$\frac{y}{b} = c - a$ $\quad \frac{y}{b} = B$

$\frac{z}{c} = a - b$ $\quad \frac{z}{c} = C$

$\therefore A + B + C = a - c + c - a - b = 0$

$\therefore A^3 + B^3 + C^3 = 3ABC$

$\therefore \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$

$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c} = \frac{3xyz}{abc}$

355. (a) $\sqrt{y} = 4x$

$\frac{x}{\sqrt{y}} = \frac{1}{4}$

Sq. both sides

$\frac{x^2}{y} = \frac{1}{16}$

356. (b) $\frac{x}{y} = \frac{a+2}{a-2}$

$\frac{x^2}{y^2} = \frac{(a+2)^2}{(a-2)^2}$

Applying Componendo & Dividendo

$\frac{x^2 - y^2}{x^2 + y^2} = \frac{(a+2)^2 - (a-2)^2}{(a+2)^2 + (a-2)^2} = \frac{8a}{2a^2 + 8} = \frac{4a}{a^2 + 4}$

357. (b) $\frac{(x + y + z)^2}{x^2 + y^2 + z^2}$

Assume $x = y = z = 1$

$\frac{(1 + 1 + 1)^3}{1 + 1 + 1} = (3)$

358. (c) $\frac{a}{b} + \frac{b}{a} = 2$

$a^2 + b^2 = 2ab$

$(a - b)^2 = 0$

$a - b = 0$

359. (a) Put $(x + y + z) = 10$

$x = 2$

$y = 3$

$z = 5$

$x(x + y + z) = 20$

$2(10) = 20$

Similarly other will satisfied so value of $2(x + y + z)$

$\Rightarrow 2(10) = 20$

360. (d) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$\Rightarrow a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$

$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$

$(a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$a - 1 = 0 \quad b + 1 = 0 \quad c + 1 = 0$

$a = 1 \quad b = -1 \quad c = -1$

$\Rightarrow a + b + c$

$\Rightarrow 1 - 1 - 1 = -1$

361. (c) $x^2 + y^2 = 14$

$x + y = 4$ ....(i)

Squaring both sides

$x^2 + y^2 + 2xy = 16$

$14 + 2xy = 16$

$2xy = 2$

$xy = 1$

$x^2 + y^2 = 14$

Substracting (2xy)from both sides.

$x^2 + y^2 - 2xy = 14 - 2xy$

$(x - y)^2 = 14 - 2 \times 1$

$x - y = \sqrt{12}$

$x - y = 2\sqrt{3}$ ....(ii)

Solve equation (i) and (ii)

$y = 2 - \sqrt{3}$

$x = 2 + \sqrt{3}$

362. (d) $x^2 - 4x - 1 = 0$

$\Rightarrow x - \frac{1}{x} = 4$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 4^2$

(squaring both sides)

$\Rightarrow x^2 + \frac{1}{x^2} = 18$

363. (c) $\left(\frac{3}{15}a^5b^6c^3 \times \frac{5}{9}ab^5c^4\right) \div \frac{10}{27}a^2bc^3$

$\Rightarrow \frac{1}{9}a^6b^{11}c^7 \div \frac{10}{27}a^2bc^3$

$\Rightarrow \frac{\frac{1}{9}a^6b^{11}c^7}{\frac{10}{27} \times a^2bc^3} \Rightarrow \frac{3}{10}a^4b^{10}c^4$

364. (b) **Given**

$\because a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, \; b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

to find $\frac{a^2}{b} + \frac{b^2}{a} = ?$

$\Rightarrow \frac{a^3+b^3}{ab} = ?$

$\Rightarrow \frac{(a+b)^3 - 3ab(a+b)}{ab} = ?$

$\Rightarrow a + b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$\Rightarrow \frac{(\sqrt{3}-\sqrt{2})^2 + (\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2 - \sqrt{2}^2}$

$\Rightarrow \frac{2(\sqrt{3}^2 + \sqrt{2}^2)}{1}$

$\Rightarrow 2 \times (5)$

$\Rightarrow a + b = 10$

Again

$\Rightarrow a \times b = \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$\Rightarrow ab = 1$

$\Rightarrow \frac{(a+b)^3 - 3ab(a+b)}{ab}$

$\Rightarrow \frac{10^3 - 3 \times 1 \times 10}{1}$

$\Rightarrow 1000 - 30 = \mathbf{970}$

365. (c) $(2a-1)^2+(4b-3)^2+(4c+5)^2 = 0$

to find

$\Rightarrow \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2}$

$\Rightarrow \frac{\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]}{a^2+b^2+c^2}$

$\Rightarrow$ then, $2a - 1 = 0$

$a = 1/2$

$\Rightarrow 4b - 3 = 0$

$\Rightarrow b = \frac{3}{4}$

$\Rightarrow 4c + 5 = 0$

$\Rightarrow c = \frac{-5}{4}$

$= a + b + c = \frac{1}{2} + \frac{3}{4} - \frac{5}{4} = 0$

$\Rightarrow \frac{\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]}{a^2+b^2+c^2}$

$= \frac{\frac{1}{2}(0)[(a-b)^2+(b-c)^2+(c-a)^2]}{a^2+b^2+c^2} = 0$

366. (b) $\frac{a}{b} = \frac{25}{6}$

$= \frac{a^2}{b^2} = \frac{25^2}{6^2}$

$\Rightarrow \frac{a^2-b^2}{a^2+b^2} = \frac{(25)^2-6^2}{(25)^2+6^2}$

$\Rightarrow \frac{625-36}{625+36} = \frac{589}{661}$

367. (a) $2x + 3y = K \quad (2=x, \; 0=y)$

$\therefore 2 \times 2 + 3 \times 0 = K$

$K = 4$

368. (a) $x + \frac{1}{x} = 3$

(Cube both sides)

Formula:-

$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$

$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$

$x^3 + \frac{1}{x^3} = 18$

$x + \frac{1}{x} = 3$ (squaring both sides)

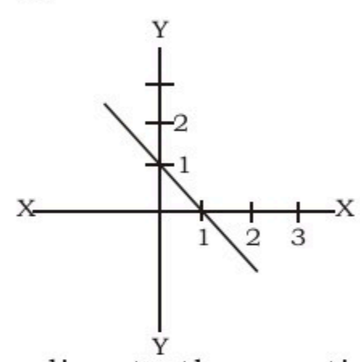
$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9$

$x^2 + \frac{1}{x^2} = 7$

Put in formula,

$x^5 + \frac{1}{x^5} = 18 \times 7 - 3 = 123$

369. (c)



According to the question,
So, option 'c' is correct.
[because y = x]

370. (a) $\left(a + \frac{1}{a}\right)^2 = 3$

$a + \frac{1}{a} = \sqrt{3}$

cube both sides

$a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = (\sqrt{3})^3$

$a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$a^3 + \frac{1}{a^3} = 0$

$a^6 + 1 = 0$

$\Rightarrow a^{30} + a^{24} + a^{18} + a^{12} + a^6 + 1$

$= a^{24}(a^6 + 1) + a^{12}(a^6+1) + a^6 + 1$

$= a^{24}(0) + a^{12}(0) + 0 = 0$

371. (d) $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$

$\frac{1}{a+b} = \frac{b+a}{ab}$

$ab = a^2 + b^2 + 2ab$

$a^2 + b^2 + ab = 0$

$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$= (a-b)(0) = 0$

372. (a) $x : y = 3 : 5$

$x - y = -2$

$\frac{x}{y} = \frac{3}{5}$

$x - y = 3 - 5 = -2 \rightarrow -2$

$x = 3, \; y = 5$

$x + y = 3 + 5 = 8$

373. (d) According to the question

$x + 1 = \sqrt{y} + 3, \; y > 0$

$\Rightarrow \frac{1}{2}\left(\frac{x^3 - 6x^2 + 12x - 8}{\sqrt{y}} - y\right) = ?$

Put $x = 4$ & $y = 4$

Now

$\Rightarrow \frac{1}{2}\left(\frac{x^3 - 6x^2 + 12x - 8}{\sqrt{y}} - y\right)$

$= \frac{1}{2}\left(\frac{(x-2)^3}{\sqrt{y}} - y\right)$

$= \frac{1}{2}\left(\frac{(4-2)^3}{\sqrt{4}} - 4\right)$

$= \frac{1}{2}\left(\frac{8}{2} - 4\right) = \frac{1}{2}(4-4) = 0$

374. (d) According to the question

$x = 1 + \sqrt{2} + \sqrt{3}$ ...(i)

$y = 1 + \sqrt{2} - \sqrt{3}$ ...(ii)

$\Rightarrow \frac{x^2 + 4xy + y^2}{x + y}$

$\Rightarrow \frac{(x+y)^2 + 2xy}{x + y}$

From equations (i) + (ii)

$x + y = 2 + 2\sqrt{2}$

$xy = \left(1+\sqrt{2}\right)^2 - \left(\sqrt{3}\right)^2$

$= 3 + 2\sqrt{2} - 3$

$= 2\sqrt{2}$

$\Rightarrow \dfrac{\left(2+2\sqrt{2}\right)^2 + 4\sqrt{2}}{2+2\sqrt{2}}$

$\Rightarrow \dfrac{4+8+8\sqrt{2}+4\sqrt{2}}{2+2\sqrt{2}}$

$\Rightarrow \dfrac{12+12\sqrt{2}}{2+2\sqrt{2}} = \dfrac{12\left(1+\sqrt{2}\right)}{2\left(1+\sqrt{2}\right)} = 6$

375. (a) $x + \dfrac{1}{x} = 3$

$x^2 + 1 = 3x$ ....(i)

$(x^2+1)^2 = 9x^2$

$x^4 + 1 + 2x^2 = 9x^2$

$x^4 + 1 = 7x^2$ ....(i)

$\therefore \dfrac{x^4+3x^3+5x^2+3x+1}{x^4+1}$

$\dfrac{12x^2+3x^3+3x}{7x^2}$

From equation (i)

$\Rightarrow \dfrac{12x+3\left(x^2+1\right)}{7x}$

$\Rightarrow \dfrac{12x+3\times 3x}{7x}$

$\Rightarrow \dfrac{21x}{7x}$

$\Rightarrow 3$

376. (c) $x + \dfrac{1}{x} = -2$

$x = -1$

$\rightarrow x^p + x^q$

$= (-1)^p + (-1)^q$

[p is even no. & q is odd no.]

$= 1 - 1 = 0$

377. (a) $p(x + y)^2 = 5$ and $q(x - y)^2 = 3$

Put the value of $x = 2$ and $y = 1$

$p(2 + 1)^2 = 5$ and $q(2 - 1)^2 = 3$

$p = \dfrac{5}{9}$ $\quad q = 3$

$\rightarrow p^2(x+y)^2 + 4pqxy - q^2(x-y)^2$

$= \left(\dfrac{5}{9}\right)^2 (2+1)^2 + 4 \times \dfrac{5}{9} \times 3 \times 2 \times 1 - (3)^2(2-1)^2$

$= \dfrac{25}{81} \times 9 + \dfrac{40}{3} - 9$

$= \dfrac{25}{9} + \dfrac{40}{3} - 9$

$= \dfrac{25+120-81}{9} = \dfrac{64}{9}$

Put the value of p and q in options (a)

option A $\rightarrow$ 2 (p + q)

$= 2\left(\dfrac{5}{9}+3\right) = 2\times \dfrac{32}{9} = \dfrac{64}{9}$

Option A is Satisfied

So, 2(p + q) is answer

378. (a) According to the question

$(p-q)^3 + (q-r)^3 + (r-p)^3$

$\Downarrow \quad \Downarrow \quad \Downarrow$

A B C

$A + B + C = p - q + q - r + r - p = 0$

If A + B + C = 0 then,

$A^3 + B^3 + C^3 = 3ABC$

$= 3 \times (p-q)(q-r)(r-p)$

379. (a) $a + \dfrac{1}{a} = \sqrt{3}$

$a^3 + \dfrac{1}{a^3} = \left(\sqrt{3}\right)^3 - 3\sqrt{3}$

$a^6 = -1$

$a^2 + \dfrac{1}{a^2} = 1$

then $a^{52} + \dfrac{1}{a^{52}} = \dfrac{a^{54}}{a^2} + \dfrac{a^2}{a^{54}}$

$= \dfrac{\left(a^6\right)^9}{a^2} + \dfrac{a^2}{\left(a^6\right)^9}$

$= -\dfrac{1}{a^2} - a^2 = -\left(a^2 + \dfrac{1}{a^2}\right)$

then $a^{52} + \dfrac{1}{a^{52}} = -1$

380. (a) $x^2 + \sigma x + \beta$

sum of root $\sigma + \beta = \dfrac{-\sigma}{1}$ ...(i)

$\sigma\ \beta = \beta$ .... (ii)

From (i) and (ii)

then $\sigma = 1$

Then $\beta = -2$

Then value of

$\sigma^3 + \beta^3 = 1 + (-2)^3 = -7$

381. (b) If $x^2 + \dfrac{1}{x^2} = 1$

Then, $x + \dfrac{1}{x} = \sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} = \left(\sqrt{3}\right)^3 - 3\sqrt{3} = 0$

$\Rightarrow x^6 = -1$, or $x^6 + 1 = 0$

then

$x^{102} + x^{96} + x^{90} + x^{84} + x^{78} + x^{72} + 5$

$x^{96}(x^6 + 1) + x^{84}(x^6 + 1) + x^{72}(x^6 + 1) + 5 = 5$

382. (c) If $x - 1$ & $x + 1$ are the factors y equation, then

$x - 1 = 0$, $x = 1$

$\Rightarrow$ Put $x = 1$, we get

$1 + a - 3 + 2 + b = 0$

$a + b = 0$ .....(i)

$\Rightarrow x + 1 = 0$, $x = -1$

Put $x = -1$, we get

$1 - a - 3 - 2 + b = 0$

$b - a = 4$ .....(ii)

after solving (i) & (ii), we get

$a = -2$, $b = 2$

383. (c) $x^3 - 7x^2 + 11x - 5 \geq 0$

$x^3 - 5x^2 - 2x^2 + 10x + x - 5 \geq 0$

$x^2(x-5) - 2x(x-5) + 1(x-5) \geq 0$

$(x-5)(x^2 - 2x + 1) \geq 0$

$(x-5)(x-1)^2 \geq 0$

$(x-5)(x-1)(x-1) \geq 0$

So, $x = 1$ & 5

Equation satisfies at both the values, but the minimum value of these two

$x = 1$

384. (a) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$676 = a^2 + b^2 + c^2 + 2(109)$

$a^2 + b^2 + c^2 = 458$

385. (c) $x^2 + \dfrac{1}{x^2} = 1$

$\Rightarrow \left(x + \dfrac{1}{x}\right)^2 = 3$

$\Rightarrow x + \dfrac{1}{x} = \sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} = 0 = x^6 + 1 = 0$

Now,

$x^{18} + x^{12} + x^6 + 1$

$x^{12}(x^6 + 1) + (x^6 + 1) = x^{12} \times 0 + 0 = 0$

386. (b) If a+b+c = 0

put a = 1 b = 1 and c = –2

$$\frac{a^2+b^2+c^2}{ab+bc+ca} = \frac{1+1+4}{1-2-2} = \frac{6}{-3} = \mathbf{-2}$$

387. (b) a+b+c = m

$$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0 \Rightarrow \frac{ab+bc+ca}{abc} = 0$$

$\Rightarrow ab+bc+ca=0$

$(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca)$

$m^2 = a^2+b^2+c^2$

$$\Rightarrow \frac{a^2+b^2+c^2}{3} = \frac{m^2}{3}$$

388. (a) $(2a+b)^2 - (2a-b)^2$

$\Rightarrow 4a^2 + b^2 + 4ab - 4a^2 - b^2 + 4ab$

$= 8ab$

389. (c) $x = \frac{8ab}{a+b}$

$$\frac{x}{4a} = \frac{2b}{a+b}$$

By componendo and dividendo

$$\frac{x+4a}{x-4a} = \frac{2b+a+b}{2b-a-b}$$

$$\frac{x+4a}{x-4a} = \frac{3b+a}{b-a} \quad \text{....(i)}$$

Similarly,

$$\frac{x+4b}{x-4b} = \frac{3a+b}{a-b} \quad \text{....(ii)}$$

adding (i) and (ii)

$$\left(\frac{x+4a}{x-4a}\right) + \left(\frac{x+4b}{x-4b}\right) = \left(\frac{3b+a}{b-a}\right) + \left(\frac{3a+b}{a-b}\right)$$

$$\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b} = \frac{-3b-a+3a+b}{a-b}$$

$$= \frac{2(a-b)}{a-b} = 2$$

390. (c) $\left(\sqrt{a}+\sqrt{b}\right) = 15$

square both side

$a + b + 2\sqrt{ab} = 225$

$a + b = 225 - 2\sqrt{ab}$ ...(i)

$\left(\sqrt{a}-\sqrt{b}\right) = 3$

Square both side

$a + b - 2\sqrt{ab} = 9$

$a + b = 9 + 2\sqrt{ab}$ ...(ii)

From (i) and (ii)

$225 - 2\sqrt{ab} = 9 + 2\sqrt{ab}$

$216 = 4\sqrt{ab}$

$54 = \sqrt{ab}$

Devided by 4 on both side

$$\frac{\sqrt{ab}}{4} = \frac{54}{4}$$

$$\frac{\sqrt{ab}}{4} = \frac{27}{2}$$

391. (d) Given $a + \frac{1}{a} = 3$

Cube both sides

$$a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a}\left(a+\frac{1}{a}\right) = (3)^3$$

$$a^3 + \frac{1}{a^3} + 3 \times 3 = 27$$

$$a^3 + \frac{1}{a^3} = 27 - 3 \times 3 = 18$$

392. (c) a – 6b = 3

after cubing both sides, we get

$\Rightarrow (a - 6b)^3 = (3)^3$

$\Rightarrow a^3 - 216\, b^3 - 18\, ab\, (3) = 27$

$\Rightarrow a^3 - 216\, b^3 - 54\, ab = 27$

393. (d) $x = 10 + 3\sqrt{11}$

By making a perfect square.

$$x = \frac{2\times(10+3\sqrt{11})}{2} = \frac{20+2\times3\sqrt{11}}{2}$$

$$x = \frac{(\sqrt{11}+3)^2}{2}$$

$$\sqrt{x} = \frac{(\sqrt{11}+3)}{\sqrt{2}}$$

then so, $\frac{1}{\sqrt{x}} = \frac{\sqrt{2}}{\sqrt{11}+3} = \frac{\sqrt{11}-3}{\sqrt{2}}$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{\sqrt{11}+3}{\sqrt{2}} - \frac{\sqrt{11}-3}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \times 6 = 3\sqrt{2}$$

394. (d) $x = \sqrt{a\sqrt[3]{ab}\sqrt{a\sqrt[3]{ab}}} \ldots\ldots\infty$

square both side

$x^2 = a\sqrt[3]{ab\,x} \quad \left(\because \sqrt{a\sqrt[3]{ab}}\ldots\ldots\infty\right)$

Again cube both sides

$x^6 = a^3\, ab\, x$

$x^5 = a^4 b$

$x = \sqrt[5]{a^4 b}$

395. (c) Put a = b = c = 1

then we have

$$\frac{m-3}{2} + \frac{m-3}{2} + \frac{m-3}{2} = 9$$

m = 9

Now, putting values of a, b & c in option,

only option (c) gives value of m = 9

So option (c) is correct.

396. (b) As both the equations leave the same remainder, then

$x - 3 = 0$

$x = 3$

Put value of $x = 3$ in both the equations & keep them equal

$a\,(3)^3 + 4(3)^2 + 3(3) - 4$

$= (3)^3 - 4(3) + a$

$27a + 36 + 9 - 4 = 27 - 12 + a$

$26a = -26$

$a = -1$

397. (c) If $(2a - 3)^2 + (3b + 4)^2 + (6c + 1)^2 = 0$

the above is possible only when every square term must be zero

then

$2a - 3 = 0$

$3b + 4 = 0$

$6c + 1 = 0$

$$a = \frac{3}{2},\ b = \frac{-4}{3}, c = \frac{-1}{6}$$

One taking a + b + c

$$= \frac{3}{2} - \frac{4}{3} - \frac{1}{6} = \frac{9-8-1}{6} = 0$$

$$\because \frac{a^3+b^3+c^3-3abc}{a^3+b^3+c^3}$$

$$= \frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{a^3+b^3+c^3}$$

$$\therefore \frac{a^3+b^3+c^3-3abc}{a^3+b^3+c^3} = 0$$

398. (c) $a^2 = b + c$, $b^2 = a + c$, $c^2 = b + c$

taking a = 2, b = 2 and c = 2

$(2)^2 = 2 + 2$

$\boxed{4 = 4}$

Now, $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}$

Put a = 2 , b = 2 and c = 2

$\frac{1}{2+1}+\frac{1}{2+1}+\frac{1}{2+1}$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$

399. (b) a + b = 2c

Taking a = 2, b = 4 and c = 3

2 + 4 = 2 × 3

$\boxed{6=6}$

Now, $\frac{a}{a-c}+\frac{c}{b-c}$

$\frac{2}{2-3}+\frac{3}{4-3}=\frac{2}{-1}+\frac{3}{1}=1$

400. (c) x + y + z = 9

$x^2 + y^2 + z^2 = 31$

$\Rightarrow (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow 81 = 31 + 2(xy + yz + zx)$

$\Rightarrow 2(xy + yz + zx) = 50$

$\Rightarrow xy + yz + zx = 25$

$\because x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zy)$

$= 9(31 - 25)$

$= 9 \times 6 = 54$

401. (a) a + b + c = 1,

ab + bc + ca = –1 and abc = –1

we know $a^2 + b^2 + c^2$

$= (a + b + c)^2 - 2(ab + bc + ca)$

then value of

$a^2 + b^2 + c^2 = (1)^2 - 2(-1) = 3$

then $a^3 + b^3 + c^3$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (1)[3 - (-1)] + 3 \times (-1) = 4 - 3 = 1$

402. (c) $\frac{a}{b}=\frac{1}{2}$ Let a = x, b = $2x$

then $\frac{(2a-5b)}{5a+3b}=\frac{2x-10x}{5x+6x}$

$=\frac{-8x}{11x}=\frac{-8}{11}$

403. (a) $3x^2 + 5x + 3 = 0$

$3x^2 + 3 = -5x$

divide by $3x$ both sides

$x+\frac{1}{x}=\frac{-5}{3}$ then $x^3+\frac{1}{x^3}$

$=\left(\frac{-5}{3}\right)^3 - 3 \times \left(\frac{-5}{3}\right)$

$=\frac{-125}{27}+5=\frac{10}{27}$

404. (b) $\frac{a}{b}+\frac{b}{a}=1$

$a^2+b^2 = ab$ ...(i)

$a^2+b^2-ab = 0$

$\because a^3+b^3 = (a+b)(a^2+b^2-ab)$ ...(ii)

From eq (i) & (ii)

$a^3 + b^3 = (a + b)\times 0$

$a^3 + b^3 = 0$

405. (a) P $(p^2 + 3p + 3)$

$= p^3 + 3p^2 + 3p + 1 - 1$

(on adding and subtraction one both side) $= (p+1)^2 - 1$

$= (99 + 1)^3 - 1 = 100^3 - 1$

$= 1000000 - 1 = 999999$

406. (b) $a^3 - b^3 = 61$

$(a - b)(a^2 + ab + b^2) = 61$

(Given a–b =1

$a^2 + ab + b^2 = 61$ ....(i)

Now,

a–b=1

On squaring both side

$a^2 + b^2 - 2ab = 1$

$\underbrace{a^2+b^2+ab}-3ab = 1$

from equ (i)

61 – 3ab =1

3ab = 60

$ab=\frac{60}{3}$

ab = 20

407. (c) $\frac{x}{a+b}+1=\frac{x}{a-b}+\frac{a-b}{a+b}$

$\frac{x}{a+b}-\frac{x}{a-b}=\frac{a-b}{a+b}-1$

$\frac{ax-bx-ax-bx}{(a+b)(a-b)}=\frac{a-b-a-b}{a+b}$

$\frac{-2bx}{(a+b)(a-b)}=\frac{-2b}{(a+b)}$

$\frac{x}{(a+b)(a-b)}=\frac{1}{(a+b)}$

$x=\frac{(a+b)(a-b)}{(a+b)}$

$x = (a - b)$

408. (c) If $x^2 + 5x + 6 = 0$

than $x^2 + 6 = -5x$

So,

$\frac{2x}{x^2-7x+6}=\frac{2x}{x^2+6-7x}$

$=\frac{2x}{-5x-7x}=\frac{2x}{-12x}=\frac{-1}{6}$

409. (a) a + b = 5

$a^2 + b^2 + 2ab = 25$ ...(i)

a – b = 3

$a^2 + b^2 - 2ab = 9$ ...(ii)

From equation (i) and (ii)

$2(a^2 + b^2) = 34$

$a^2 + b^2 = 17$

410. (c) $x = a + \frac{1}{a}$

$x^2 = a^2 + \frac{1}{a^2} + 2$

$y^2 = a^2 + \frac{1}{a^2} - 2$

Now,

$x^4 + y^4 - 2x^2y^2 = (x^2 - y^2)^2$

$=(a^2+\frac{1}{a^2}+2-a^2-\frac{1}{a^2}+2)^2$

$= (4)^2 = 16$

411. (d) $2^{x+4} - 2^{x+2} = 3$

$2^x.2^4 - 2^x.2^2 = 3$

$2^x(2^4 - 2^2) = 3$

$2^x(16 - 4) = 3$

$2^x=\frac{3}{12}=\frac{1}{4}$

$2^x=\frac{1}{2^2}=2^{-2}$

$2^x = 2^{-2}$

$x = -2$

412. (b) $x+\frac{1}{x}=\sqrt{3}$

Cubing both side

$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=3\sqrt{3}$

$x^3+\frac{1}{x^3}+3\left(\sqrt{3}\right)=3\sqrt{3}$

$x^3+\frac{1}{x^3}=3\sqrt{3}-3\sqrt{3}=0$

413. (b) $a+\frac{1}{a}=2$

put a = 1 (satisfy)

$a^{109}+\frac{1}{a^{109}}=1^{109}+\frac{1}{1^{109}}=1+1=2$

414. (a) $2x-\frac{1}{2x}=5$

divide by 2 both side

$x-\frac{1}{4x}=\frac{5}{2}$

Squaring both side

$x^2+\frac{1}{16x^2}-2\times x\times\frac{1}{4x}=\frac{25}{4}$

$x^2+\frac{1}{16x^2}-\frac{1}{2}=\frac{25}{4}$

$x^2+\frac{1}{16x^2}=\frac{25}{4}+\frac{1}{2}=\frac{27}{4}$

So,

$x^2+\frac{1}{16x^2}-2=\frac{27}{4}-2$

$x^2+\frac{1}{16x^2}-2=\frac{19}{4}$

415. (c) Given that $x=\frac{1}{\sqrt{2}+1}$

and x^2+2x-1

$=x^2+2x-1+1-1$

$=x^2+2x+1-2$

$=(x+1)^2-2$

Now put the value of x

$=\left(\frac{1}{\sqrt{2}+1}+1\right)^2-2$

$=\left(\frac{1+\sqrt{2}+1}{\sqrt{2}+1}\right)^2-2$

$=\left(\frac{\sqrt{2}+2}{\sqrt{2}+1}\right)^2-2$

$=\left(\frac{(\sqrt{2}+1)\times\sqrt{2}}{\sqrt{2}+1}\right)^2-2$

$=\left[\sqrt{2}\right]^2-2=2-2=0$

416. (c) Given $=x+\frac{1}{x}=\sqrt{13}$

than $=\frac{3x}{x^2-1}=\frac{3}{x-\frac{1}{x}}$ ....(i)

Now $x+\frac{1}{x}=\sqrt{13}$

on squaring both side

$x^2+\frac{1}{x^2}=13-2=11$

$x^2+\frac{1}{x^2}-2=11-2$

$\left(x-\frac{1}{x}\right)^2=9=3^2$

$x-\frac{1}{x}=3$

Put this value in equation (i)

$=\frac{3}{x-\frac{1}{x}}=\frac{3}{3}=1$

417. (d) a(x + y) = b(x – y) = 2ab

a(x + y) = 2ab

x + y = 2b

On squaring both side

$x^2+y^2+2xy=4b^2$ ....9(i)

b (x – y) = 2ab

$x-y=2a$

on squaring both side

$x^2+y^2-2xy=4a^2$ ....(ii)

add equation (i) + (ii)

$2(x^2+y^2)=4(a^2+b^2)$

418. (c) $x+\frac{1}{x}=6$ on squaring both side

$x^2+\frac{1}{x^2}=36-2$

$x^2+\frac{1}{x^2}=34$

419. (b) $C+\frac{1}{C}=3$

$C-3+\frac{1}{C}=0$

So $(C-3)^7+\frac{1}{C^7}=\left[(C-3)+\frac{1}{C}\right]^3$

$\left[(C-3)+\frac{1}{C}\right]^4-\left[(C-3)+\frac{1}{C}\right]=0$

420. (a) $2x+\frac{1}{4x}=1$

Dividing by 2 both side

$x+\frac{1}{8x}=\frac{1}{2}$

Squaring both side

$x^2+\frac{1}{64x^2}+2\times x\times\frac{1}{8x}=\frac{1}{4}$

$x^2+\frac{1}{64x^2}=\frac{1}{4}-\frac{1}{4}=0$

421. (b) $\sqrt{x}-\sqrt{y}=1$ ....(i)

$\sqrt{x}+\sqrt{y}=17$ ....(ii)

From eqn (i) & eq (ii)

$\sqrt{x}=9,\ \sqrt{y}=8$

So $\sqrt{xy}=9\times8=72$

422. (c) $x=\sqrt{3}+\frac{1}{\sqrt{3}}=\frac{3+1}{\sqrt{3}}=\frac{4}{\sqrt{3}}$

So $\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)$

$=\left(\frac{4}{\sqrt{3}}-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{\frac{4}{\sqrt{3}}-\frac{2}{\sqrt{3}}}\right)$

$=\left(\frac{4\sqrt{42}-\sqrt{126}\times\sqrt{3}}{\sqrt{3}\times\sqrt{142}}\right)\left(x-\frac{1}{2/\sqrt{3}}\right)$

$=\frac{4\sqrt{42}-3\sqrt{42}}{\sqrt{3}\times\sqrt{42}}\left(x-\frac{\sqrt{3}}{2}\right)$

$=\frac{4\sqrt{42}-3\sqrt{42}}{\sqrt{3}\times\sqrt{42}}\left(\frac{4}{\sqrt{3}}-\frac{\sqrt{3}}{2}\right)$

$=\left(\frac{\sqrt{42}}{\sqrt{3}\times\sqrt{42}}\right)\left(\frac{8-3}{2\sqrt{3}}\right)=\frac{5}{6}$

423. (b) $a^3-b^3=(a-b)(a^2+b^2+ab)$

$56=2(a^2+b^2+ab)$

$28=a^2+b^2+ab$ ....(i)

$a-b=2$

$a^2+b^2-2ab=4$ ....(ii)

From equation (i) and eq. (ii)

$\boxed{a^2+b^2=20}$

OR,

put a = 4, b = 2, a – b = 4 – 2 = 2 (satisfy)

So, $a^2+b^2=16+4=20$

424. (c) $x^2 - 3x + 1 = 0$

Divide by x

$$x - 3 + \frac{1}{x} = 0$$

$$\boxed{x + \frac{1}{x} = 3}$$

425. (d) $\frac{2+a}{a} + \frac{2+b}{b} + \frac{2+c}{c} = 4$

$$\frac{2bc + abc + 2ac + abc + 2ab + abc}{abc} = 4$$

$$\frac{2(bc+ab+ca)}{abc} + \frac{3abc}{abc} = 4$$

$$\frac{2(bc+ab+ca)}{abc} + 3 = 4$$

$$\boxed{\frac{ab+bc+ca}{abc} = \frac{1}{2}}$$

426. (b) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

$$\frac{xy+yz+zx}{xyz} = 1$$

$= xy + yz + zx = xyz = -1$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2\ (xy + yz + zx)$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2\ (xy + yz + zx)$

$(1)^2 = x^2 + y^2 + z^2 + 2(-1)$

$x^2 + y^2 + z^2 = 3$

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)\ [x^2 + y^2 + z^2 - (xy + yx + zx)$

$x^3 + y^3 + z^3 - 3\ (-1) = 1[3 - (-1)]$

$x^3 + y^3 + z^3 = 4 - 3 = 1$

427. (c) $x + \frac{1}{x} = 5$ then $\frac{5x}{x^2+5x+1}$

$$= \frac{5}{x+5+\frac{1}{x}} = \frac{5}{x+\frac{1}{x}+5}$$

$$= \frac{5}{5+5} = \frac{1}{2}$$

428. (b) $\frac{1}{a}\ (a^2+1) = 3$

$a + \frac{1}{a} = 3$

$a^3 + \frac{1}{a_3} = (3)^3 - 3\times3$

$= 27 - 9 = 18$

$\frac{a^6+1}{a_3} = 18$

429. (b) $a = \sqrt{2} + 1,\ b = \sqrt{2} - 1$

$$\frac{1}{a+1} + \frac{1}{b+1} = \frac{1}{\sqrt{2}+1+1} +$$

$$\frac{1}{\sqrt{2}-1+1} = \frac{1}{\sqrt{2}+2} + \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}(\sqrt{2}+1)} + \frac{1}{\sqrt{2}}$$

$$= \frac{1+\sqrt{2}+1}{\sqrt{2}(\sqrt{2}+1)} = \frac{2+\sqrt{2}}{2+\sqrt{2}} = 1$$

430. (c) $(x-5)^2 + (y-2)^2 + (z-9)^2 = 0$

It is possible only when $x - 5 = 0$

$x = 5$

$y - 2 = 0$

$y = 2$

$z - 9 = 0$

$z = 9$

$\therefore x + y - z = 5 + 2 - 9$

$= 7 - 9 = -2$

431. (c) $x + \frac{1}{x} = 3$

On squaring both side, we get

$$x^2 + \frac{1}{x^2} = 7$$

Again, On squaring both side

$$x^4 + \frac{1}{x^4} = 47$$

And again squaring both side.

$$x^8 + \frac{1}{x^8} = 2207$$

432. (d) $\because a^3 + b^3 + c^3 - 3abc$

$= \frac{1}{2}(a + b + c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$$\therefore \frac{x^3+y^3+z^3-3xyz}{x-y+z}$$

$$= \frac{\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-2)^2+(z-x)^2\right]}{x-y+z}$$

$$= \frac{\frac{1}{2}(999+1000+1001)[1+1+4]}{999-1000+1001}$$

$$= \frac{\frac{1}{2}\times6\times3000}{1000} = 9$$

433. (d) $\frac{a}{1-2a} + \frac{b}{1-2b} + \frac{c}{1-2c} = \frac{1}{2}$

Multiply by 2 both side

$$\Rightarrow \frac{2a}{1-2a} + \frac{2a}{1-2b} + \frac{2c}{1-2c} = 1$$

Adding 3 both side

$$\Rightarrow 1 + \frac{2a}{1-2a} + 1 + \frac{2b}{1-2b} + 1$$

$$\frac{2c}{1-2c} = 1 + 3$$

$$\Rightarrow \frac{1}{1-2a} + \frac{1}{1-2b} + \frac{1}{1-2c} = 4$$

434. (c) Let numbers are

$x = 2r,\ y = 3r,\ z = 5r$

Sum, $x + y + z = 2r + 3r + 5r$

$= 10r = 80$

$r = 8,\ x = 16,\ y = 24,\ z = 40$

Then $z = ax - 8$

$40 = a \times 16 - 8$

$a = 3$

435. (c) $(x - 2)\ (x - p) = x^2 - ax + 6$

$x^2 - (2 + p)\ x + 2p = x^2 - ax + 6$

Comparision the cofficients

$2 + p = a,\ 2p = 6$

$p = 3$

$2 + 3 = a = 5$

then, $p = 3,\ a = 5$

$a - p = 5 - 3 = 2$

436. (a) $x = \sqrt{a} + \frac{1}{\sqrt{a}},\ y = \sqrt{a} - \frac{1}{\sqrt{a}}$

put $a = 4$

$$x = 2 + \frac{1}{2} = \frac{5}{2},\ y = 2 - \frac{1}{2} = \frac{3}{2}$$

then, $x^4 + y^4 - 2x^2y^2$

$= (x^2 - y^2)^2$

$$= \left(\frac{25}{4} - \frac{9}{4}\right)^2 = 16$$

437. (b) $x = \sqrt[3]{x^2+11} - 2$

$x + 2 = \sqrt[3]{x^2+11}$

$\Rightarrow$ Taking cube on both side

$(x + 2)^3 = x^2 + 11$

$x^3 + 8 + 6x\ (x + 2) = x^2 + 11$

$x^3 + 8 + 6x^2 + 12x = x^2 + 11$

$x^3 + 5x^2 + 12x = 3$

438. (c) $P^2 + \frac{1}{P^2} = 47$

on adding 2 both side

$$P^2 + \frac{1}{P^2} + 2 = 47 + 2$$

$$(P + \frac{1}{P})^2 = 49$$

$$(P + \frac{1}{P}) = 7$$

$$\mathbf{P + \frac{1}{P} = 7}$$

439. (c) Third ratio

a : x : : x : b

x^2 = ab

$x = \sqrt{(x-y)^2(x^2-y^2)^2}$

$x = (x - y)\ (x^2 - y^2)$

$x = (x - y)\ (x - y)\ (x + y)$

$x = (x + y)\ (x - y)^2$

440. (a) $x + \sqrt{5} = 5 + \sqrt{y}$

put, $x = 5$ and y = 5

$5 + \sqrt{5} = 5 + \sqrt{5}$

L.H.S. = R.H.S

$$\frac{\sqrt{x}+y}{x+\sqrt{y}} = \frac{\sqrt{5}+5}{5+\sqrt{5}} = 1$$

441. (d) This is possible only when

$(x - 3)^2 = 0$

$x = 3$,

$(y - 4)^2 = 0$

y = 4,

$(z - 5)^2 = 0$

z = 5

then = x +y + z = 3 + 4 + 5 = 12

442. (d) $2x + \frac{1}{3x} = 5$

$6x^2 + 1 = 15x$ ....(i)

Now,

$$= \frac{5x}{6x^2+20x+1} = \frac{5x}{6x^2+1+20x}$$

$$= \frac{5x}{15x+20x} \text{ (From equation (i))}$$

$$= \frac{5x}{35x} = \frac{1}{7}$$

443. (b) a + b = 10 and ab = 21

(a + b) = 10

$a^2 + b^2 + 2ab = 100$

$a^2 + b^2 = 100 - 2ab$

$a^2 + b^2 = 100 - 2 \times 21$

= 100 – 42

$a^2 + b^2 = 58$ ....(i)

$(a - b)^2 = a^2 + b^2 - 2ab$

$= 58 - 2 \times 21$

(from equation (i))

= 58 – 42

$(a - b)^2 = 16$

444. (c) We know that

$a^3 - b^3 = (a - b)\ (a^2 + ab + b^2)$

$x^3 - P = (x - 4)\ (x^2 + 4x + 16)$

$x^3 - P = x^3 - 4^3$

$P = 4^3$ (By comparison)

So, P = 64

445. (b) $\Rightarrow \frac{5x}{4x^2+10x+1}$

$$= \frac{5x}{x\left(4x+10x+\frac{1}{x}\right)}$$

$$= \frac{5}{4x+\frac{1}{x}+10} = \frac{5}{5+10} = \frac{5}{15} = \frac{1}{3}$$

446. (a) $C + \frac{1}{C} = \sqrt{3}$

On cubing both side

$$\left(C+\frac{1}{C}\right)^3 = 3\sqrt{3}$$

$$C^3 + \frac{1}{C^3} + 3 \times C \times \frac{1}{C}\left(C+\frac{1}{C}\right) = 3\sqrt{3}$$

$$C^3 + \frac{1}{C^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$C^3 + \frac{1}{C^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

447. (b) $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\ [(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(222 + 223 + 225)\ [(222 - 223)^2 + (223 - 225)^2 + (225 - 222)^2]$

$= \frac{1}{2}(670)[1 + 4 + 9]$

$= \frac{1}{2} \times 670 \times 14 = 4690$

448. (c) a + b + c = 0

$a^3 + b^3 + c^3 = T$

$a^3 + b^3 + c^3 - 3abc$

$= \frac{1}{2}(a + b + c)\ [(a - b)^2 + (b - c)^2 + (c - a)^2]$

$a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$

449. (d) $\left(x^{\frac{1}{3}} + x^{\frac{-1}{3}}\right)\left(x^{\frac{2}{3}} + x^{\frac{-2}{3}} - 1\right)$

We know that

$(a^3 + b^3) = (a + b)\ (a^2 + b^2 - ab)$

$$\left(x^{\frac{1}{3}\times 3} + x^{\frac{1}{x}} \times 3\right)$$

$(x + x^{-1})$

Alternative,

$$\left(x^{\frac{1}{3}} + \frac{1}{x^{\frac{1}{3}}}\right)\left(x^{\frac{2}{3}} - 1\frac{1}{x^{\frac{2}{3}}}\right)$$

$$\left(x^{\frac{1}{3}} + \frac{1}{x^{\frac{1}{3}}}\right)\left(x^{\frac{2}{3}} + \frac{1}{x^{\frac{2}{3}}} - 1\right)$$

$$x^{\frac{1}{3}} \cdot x^{\frac{2}{3}} + x^{\frac{1}{3}} \cdot \frac{1}{x^{\frac{2}{3}}} - x^{\frac{1}{3}} + x^{\frac{2}{3}} \cdot$$

$$\frac{1}{x^{\frac{1}{3}}} + \frac{1}{x^{\frac{2}{3}}x^{\frac{1}{3}}} - \frac{1}{x^{\frac{1}{3}}}$$

$$x + x^{\frac{-1}{3}} - x^{\frac{1}{3}} + x^{\frac{1}{3}} + \frac{1}{x} - x^{\frac{-1}{3}}$$

$x + x^{-1}$

450. (d) $\frac{1}{P} + \frac{1}{q} = \frac{1}{p+q}$

$$\frac{p+q}{pq} = \frac{1}{p+q}$$

$(p + q)^2 = pq$

$p^2 + q^2 + 2pq - pq = 0$

$(p^2 + q^2 + pq) = 0$

Multiply by (p – q) both side

$(p - q)\ (p^2 + q^2 + pq) = (p-q)\times 0$

$p^3 - q^3 = 0$

451. (c) $x^2 - y^2 + 10xz + 10yz$

$(x-y)(x+ y) + 10z(x + y)$

Put value of x, y, and z

$\Rightarrow (93 - 93)\ (93 + 93) + 10 \times 94$

(93 + 93)

$0 + 940 \times 186$

$\Rightarrow 174840$

452. (b) $\frac{(a+b)^2}{(a-b)^2} = \frac{25}{4}$

$$\frac{a+b}{a-b} = \frac{5}{2}$$

by C & D

$$\frac{a+b+a-b}{a+b-a+b} = \frac{5+2}{5-2}$$

$$\frac{2a}{2b} = \frac{7}{3}$$

$$\frac{a}{b} = \frac{7}{3}$$

Now, the value of

$\Rightarrow a^2 + b^2 + 3ab$

$= 7^2 + 3^2 + 3 \times 7\times 3$

= 49 + 9 + 63 = 58 + 63

= 121

453. (c) $a - b = 2$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$26 = 2(a^2 + ab + b^2)$

$13 = a^2 + ab + b^2$ ....(i)

$4 = 13 + ab$

$(a - b)^2 = a^2 + b^2 - 2ab$

$(2)^2 = a^2 + b^2 + ab - 3ab$

$3ab = 9$

$ab = 3$

$(a + b)^2 = (a - b)^2 + 4ab$

$= 4 + 4 \times 3 = 16$

454. (c) $(x - 4)^3 + (y - 2)^3 + (z - 3)^3$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$a^3 \quad b^3 \quad c^3 - 3abc$

$-3(x - 1)(y - 2)(z - 3)$

$a + b + c = x - 4 + y - 2 + z - 3$

$= x + y + z - 9$

$= 9 - 9 = 0$

$a + b + c = 0$

So, $(x - 4)^3 + (y-2)^3 + (z-3)^3 - 3(x - 4)(y - 2)(z - 3) = 0$

455. (b) $x + \frac{1}{9x} = 4$

Multiply by 3 both side

$3x + \frac{1}{3x} = 12$

squaring both side

$9x^2 + \frac{1}{9x^2} + 2 \times 3x \times \frac{1}{3x} = 144$

$9x^2 + \frac{1}{9x^2} + 2 = 144$

$9x^2 + \frac{1}{9x^2} = 142$

456. (d) $a = 2, b = -3$

So, $= 27a^3 + 36ab^2 - 8b^3 - 54a^2b$

$= 27 \times (2)^3 + 36 \times 2 \times (-3)^2 - 8 \times (-3)^3 - 54 \times (2)^2 \times (-3)$

$= 27 \times 8 + 36 \times 2 \times 9 + 8 \times 27 + 54 \times 4 \times 3$

$= 216 + 648 + 216 + 648 = 1728$

457. (b) but $a = 1$

$a^2 + \frac{1}{a^3} = 2, 1^3 + \frac{1}{1^3} = 2$

(satisfy)

So, $\frac{a^2+1}{a} = \frac{1^2+1}{1} = 2$

458. (d) $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$

$3x - 2 = \frac{3}{x}$

$3x - \frac{3}{x} = 2$

$3\left(x - \frac{1}{x}\right) = 2$

$x - \frac{1}{x} = \frac{2}{3}$

Squaring both side

$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = \frac{4}{9}$

$x^2 + \frac{1}{x^2} = \frac{4}{9} + 2$

$\left[x^2 + \frac{1}{x^2} = 2\frac{4}{9}\right]$

$x^2 + \frac{1}{x^2} = 2\frac{4}{9}$

459. (a) Let $\frac{a}{q-r} = \frac{b}{r-p} = \frac{c}{p-q} = k$

$\frac{a}{q-r} = k$ (on multiplying by P)

$pa = k(pq-pr)$ .....(i)

In the same way we can write

$qb = k(qr-qp)$ .....(ii)

and $rc = k(rp-rq)$ ...(iii)

on adding eq (i), (ii) and (iii)

$pa + qb + rc = k(pq - pr + qr - qp + rp - rq)$

$pa + qb + rc = 0$

460. (a) $\left(a + \frac{1}{a}\right)^2 = 3$

$a + \frac{1}{a} = \sqrt{3}$

Taking cube on both sides

$\Rightarrow a^3 + \frac{1}{a^3} + 3(a) \times \left(\frac{1}{a}\right) \times \left(a + \frac{1}{a}\right)$

$= 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

461. (d) $\frac{a^2+b^2}{c^2} = \frac{b^2+c^2}{a^2} = \frac{c^2+a^2}{b^2} = \frac{1}{k}$

Put a=b=c = 1

$\frac{1+1}{1} + \frac{1+1}{1} + \frac{1+1}{1} = \frac{1}{k}$

$2 = 2 = 2 = \frac{1}{k}$

$k = \frac{1}{2}$

462. (b) $2x + \frac{2}{9x} = 4$

Multiply by $\frac{3}{2}$ on both sides

$3x + \frac{1}{3x} = 6$

Taking cube on both sides

$\left(3x + \frac{1}{3x}\right)^2 = (6)^3$

$27x^3 + \frac{1}{27x^3} + 3 \times 3x \times \frac{1}{3x}$

$\left(3x + \frac{1}{3x}\right) = 216$

$27x^3 + \frac{1}{27x^3} + 3 \times 6 = 216$

$27x^3 + \frac{1}{27x^3}$

$= 216 - 18 = 198$

463. (b) $x^2 + \frac{1}{x^2} = 2$

$\left(x - \frac{1}{x}\right)^2 + 2 = 2$

$x - \frac{1}{x} = 0$

464. (c) $pq(p + q) = 1$

$p + q = \frac{1}{pq}$

$p3 + q3 + 3pq(p + q) = \frac{1}{p^3q^3}$

$\frac{1}{p^3q^3} - p^3 - q^3 = \left(\frac{13}{p+q}\right)(p+q)$

$= 3$

465. (d) $x = \sqrt{2} + 1, \frac{1}{x} = \sqrt{2} - 1$

$x - \frac{1}{x} = 2$

$x + \frac{1}{x} = 2\sqrt{2}$

$x^2 + \frac{1}{x^2} = 6$

$x^4 - \frac{1}{x^4} = \left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)$

$= \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)$

$= 2 \times 2\sqrt{2} \times 6 = 24\sqrt{2}$

466. (a) $p^3 - q^3 = (p - q)(p^2 + q^2 + pq)$

$(p - q)\{(p + q)^2 - xpq\} = (p - q)(p^2 + q^2 + pq)$

$p^2 + q^2 + 2pq - xpq = p^2 + q^2 + pq$

$2pq - pq = xpq$

$pq = xpq$

$\therefore\ x = 1$

467. (d) $x = \sqrt[2]{0.25} = 0.5$

$y = (0.4)^2 = 0.16$

$z = (0.216)^{1/2} = 0.464$

$\therefore\ x > z > y$

468. (a) $a^2 = by + cz$

$b^2 = cz + ax$

$c^2 = ax + by$

$$= \frac{x}{(a+x)} + \frac{y}{(b+y)} + \frac{z}{(c+z)}$$

$$= \frac{ax}{a^2+ax} + \frac{by}{b^2+by} + \frac{cz}{c^2+cz}$$

$$= \frac{ax}{by+cz+ax} + \frac{by}{cz+ax+by} + \frac{cz}{ax+by+cz} = \frac{ax+by+cz}{by+cz+ax} = 1$$

469. (b) $(x + y)^2 = x^2 + y^2 + 2xy$

$= 29 + 2 \times 10$

$= 49$

$x + y = 7$

$(x - y)^2 = x^2 + y^2 - 2xy$

$= 29 - 2 \times 10 = 9$

$(x - y) = 3$

$$\frac{x+y}{x-y} = \frac{7}{3}$$

470. (a) If $\frac{a}{b} + \frac{b}{a} = 1$

than, $a^2 + b^2 = ab$

$a^2 + b^2 - ab = 0$

and $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$= (a + b)(0)$

$a^3 + b^3 = 0$

471. (c) $p^2 + q^2 = 7pq$.

$$\frac{p}{q} + \frac{q}{p} = 7$$

(divide whole eq^n by pq)

472. (a) $x + 3^{-\frac{1}{3}} = 3^{\frac{1}{3}}$

$$x^3 + \frac{1}{3} + 3 \times x3^{-\frac{1}{3}}\left(x + 3^{-\frac{1}{3}}\right) = 3$$

$$x^3 + \frac{1}{3} + 3 \times x3^{-\frac{1}{3}} \times 3^{\frac{1}{3}} = 3$$

$3x^3 + 9x = 9 - 1 = 8$

473. (c) $xy(x + y) = m$, find $x^3 + y^3 + 3m$

$xy(x + y) = m$ ....(i)

$$(x + y) = \frac{m}{xy}$$

Cubing both side

$$x^3 + y^3 + 3 \cdot xy(x + y) = \frac{m^3}{x^3y^3}$$

from equation (i)

$$x^3 + y^3 + 3 \times m = \frac{m^3}{x^3y^3}$$

$$x^3 + y^3 + 3m = \frac{m^3}{x^3y^3}$$

474. (d) $P + \frac{1}{P+2} = 1$, Find the value of $(P + 2)^3 + \frac{1}{(P+2)^3} - 3$

Adding 2 on both sides

$$(P + 2) + \frac{1}{(P+2)} = 3$$

Taking cube on both sides

$$(P + 2)^3 + \frac{1}{(P+2)^3} + 3(P+2) \times \frac{1}{(P+2)}$$

$$\left[(P+2) + \frac{1}{P+2}\right] = \left((P+2) + \frac{1}{(P+2)}\right)^3 = 3^3 = 27$$

$$(P + 2)^3 + \frac{1}{(P+2)^3} + 9 = 27$$

$$(P+2)^3 + \frac{1}{(P+2)^3} = 18$$

According to questions

$18 - 3 \Rightarrow 15$

475. (a) $x^3 + \frac{1}{x^3} = 0$

It is possible only when

$$x + \frac{1}{x} = \sqrt{3}$$

$$\text{So, } \left(x + \frac{1}{x}\right)^4 = \left(\sqrt{3}\right)^4 = 9$$

476. (b) $x + \frac{1}{x} = 2$

But $x = 1$

$$\text{So, } x^2 + \frac{1}{x^2} = 1^2 + \frac{1}{1^2} = 2$$

477. (c) $x + y = \sqrt{3}$ ....(i)

$x - y = \sqrt{2}$ ....(ii)

From eq (i) and (ii)

$$x = \frac{\sqrt{3}+\sqrt{2}}{2}, y = \frac{\sqrt{3}-\sqrt{2}}{2}$$

So,

$8xy(x^2 + y^2)$

$$= 8 \times \frac{\left(\sqrt{3}+\sqrt{2}\right)}{2} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)}{2}$$

$$\left[\frac{\left(\sqrt{3}+\sqrt{2}\right)^2}{4} + \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{4}\right]$$

$$= 2(3 - 2)\left[\frac{3+2+2\sqrt{6}+3+2-2\sqrt{6}}{4}\right]$$

$$= 2 \times 1 \times \frac{10}{4} = 5$$

478. (b)

$$\frac{x + \frac{1}{x}}{x - \frac{1}{x}} = \frac{5}{3}$$

By C & D

$$\frac{x + \frac{1}{x} + x - \frac{1}{x}}{x + \frac{1}{x} - x + \frac{1}{x}} = \frac{5+3}{5-3}$$

$$\frac{2x}{\frac{2}{x}} = \frac{8}{2} = 4$$

$x^2 = 4$

$x = \pm 2$

479. (c) $p^3 - q^3 = (p - q)[(p - q)^2 + xpq]$

$(p - q)(p^2 + q^2 + pq) = (p - q)[p^2 + q^2 - 2bq + xpq]$

$\Rightarrow p^2 + q^2 + pq = p^2 + q^2 - 2pq + xpq$

$3pq = xpq$

$x = 3$

480. (c) $\left(a + \frac{1}{a}\right)^2 = 3$

$$a + \frac{1}{a} = \sqrt{3}$$

$$a^3 + \frac{1}{a^3} = 0$$

$a^6 = -1$

$(a^6 + 1) = 0$

so $a^{18} + a^{12} + a^6 + 1$

$\Rightarrow a^{12}(a^6 + 1) + a^6 + 1$

$\Rightarrow a^{12} \times 0 + 0$

$\Rightarrow 0$

481. (d) We know that

$$(a^2 + b^2 + c^2 - ab - bc - ca) = \frac{1}{2}$$

$$\left[(a-b)^2 + (b-c)^2 + (c-d)^2\right]$$

$$\therefore \begin{bmatrix} 113^2 + 115^2 + 117^2 - 113 \times 115 \\ -115 \times 117 - 113 \times 117 \end{bmatrix}$$

$= \frac{1}{2}[(113 - 115)^2 + (115 - 117)^2 + (117 - 113)^2]$

$= \frac{1}{2}[4 + 4 + 16] = 12$

482. (a) Let the quotient is y.

divisor)Number(quotient

Remainder

4y)Number(y

2y ← Rema

2y = 80

y = 40

160)Number(40

80

Number = 160 × 40 + 80 = 6480

483. (b) P = 99

P (P² + 3P + 3)

= P³ + 3P² + 3P + 1 – 1

= (P+1)³ – 1 = (99 + 1)³–1

= (100)³ – 1 = (100)³–1

= 1000000–1 = 999999

484. (a) If $n + \frac{1}{n} = c + \frac{1}{c}$

go through the option

put n = c, $\frac{1}{c}$

if n = c

$c + \frac{1}{c}$ = satisfied

if $n = \frac{1}{c}$

then $n + \frac{1}{n}$

$\frac{1}{c} + \frac{1}{1/c} = \frac{1}{c} + c$ satisfied

485. (c) We no. are a,b

$a^2 + b^2 = 41$

a + b = 9

$(a+b)^2 = a^2+b^2+2ab$

$a^2 = 41 + 2ab$

81 = 41 + 2ab

a.b = 20

take = a = 5

b = 4

$a^3+b^3 = 5^3 + 4^3$

= 125 + 64 = 189

486. (d) $(x^4 + 64)$

$= x^4 + 8^2 + 2.x^2.8 - 2x^2.8$

$= (x^2 + 8)^2 - (16x^2)$

$= (x^2 + 8)^2 - (4x)^2$

$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$

487. (d) If a+b = 1

$a^4 + b^4 - a^3 - b^3 - 2a^2b^2 + ab$

(a + b) = 1

$(a + b)^3 = 1$

$a^3 + b^3 + 3ab(a + b) = 1$

$a^3 + b^3 + 3ab = 1$

$-a^3 - b^3 = -1 + 3ab$

$a^4 + b4 - 2a^2b^2 - a^3 - b^3 + ab$

$(a^2 - b^2)^2 - 1 + 3ab + ab$

$((a + b)(a - b))^2 - 1 + 4ab$

$(a-b)^2 - 1 + 4ab$

$a^2+b^2 - 2ab + 4ab$

$a^2+b^2 - 2ab + 4ab-1$

$(a^2 + b^2 + 2ab) - 1 = (a + b)^2 - 1$

= 1 – 1 = 0

488. (a) $x^2 + y^2 + 6x + 5 = 4(x - y)$

$x^2 + y^2 + 6x + 5 - 4x + 4y = 0$

$x^2 + 2x + 1 + y^2 + 4y + 4 = 0$

$(x + 1)^2 + (y + 2)^2 = 0$

0 0

x + 1 = 0

x = – 1

y + 2 = 0

y = – 2

(x – y) = –1 – (–2) = –1 + 2 = 1

489. (c) a = 299

b = 298

c = 297

$2a^3 + 2b^3 + 2c^3 - 6abc$

$= 2(a^3 + b^3 + c^3 - 3abc)$

$= 2(a+b+c)\ [a^2+b^2+c^2-ab-bc-ca)]$

$= 2 \times \frac{1}{2}(a+b+c)$

$[(a-b)^2+(b-c)^2+(c-a)^2]$

$= (a+b+c)\ [(a-b)^2+(b-c)^2+(c-a)^2]$

$= (299 + 298 + 297)\ [(299 - 298)^2$

$+ (298 - 297)^2 + (297-299)^2$

$= 894 \times [1^2+1^2+2^2]$

$894 \times [1+1+4]$

$= 894 \times 6 = 5364$

490. (a) $x + \frac{1}{x} = \sqrt{3}$

$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$= 3\sqrt{3} - 3\sqrt{3} = 0$

$x^3 + \frac{1}{x^3} = 0$

$x^6 + 1 = 0$

$x^{18} + x^{12} + x^6 + 1$

$x(x^6 + 1) + (x^6 + 1)$

$x^{12} \times 0 + 0 = 0$

491. (a) $x = 1 + \sqrt{2} + \sqrt{3}$

$x - 1 = \sqrt{2} + \sqrt{3}$

Squaring both side

$x^2 - 2x + 1 = 2 + 3 + 2\sqrt{6}$

$x^2 - 2x - 4 = 2\sqrt{6}$

Squaring on both sides

$(x^2 - 2x - 4)^2 = (2\sqrt{6})^2$

$x^4 + (2x + 4)^2 - 2x^2(2x + 4) = 24$

$x^4 - 4x^3 + 4x^2 - 8x^2 + 16x + 16 = 24$

$x^4 - 4x^3 - 4x^2 + 16x = 8$

$2x^4 - 8x^3 - 8x^2 + 32x = 16$

$2x^4 - 8x^3 - 5x^2 + 26x - 3(x^2 - 2x) = 16$

$2x^4 - 8x^3 - 5x^2 + 26x - 28 = 16 -$

$28 + 3(2\sqrt{6} + 4)$

$2x^4 - 8x^3 - 5x^2 + 26x - 28 = 16 -$

$28 + 12 + 6\sqrt{6}$

$2x^4 - 8x^3 - 5x^2 + 26x - 28 = 6\sqrt{6}$

492. (c) $2r = h + \sqrt{r^2+h^2}$

$(2r-h) = \sqrt{r^2+h^2}$

$(2r-h)^2 = \left(\sqrt{r^2+h^2}\right)^2$

$4r^2 + h^2 - 4rh = r^2+h^2$

$3r^2 = 4rh$

$\frac{r}{h} = \frac{4}{3}$

493. (b) $\left(ab\left(1 - \frac{2xy}{x^2+y^2}\right) \div \left(\frac{x^3-y^3}{x-y} - 3xy\right)\right.$

$= \left(\frac{x^2+y^2-2xy}{x^2+y^2}\right) \div \left[\frac{x^3-y^3-3xy(x-y)}{x-y}\right]$

$= \frac{(x-y)^2}{x^2+y^2} \div \frac{(x-y)^3}{x-y}$

$= \frac{(x-y)^2}{x^2+y^2} \div (x-y)^2$

$= \frac{(x-y)^2}{x^2+y^2} \times \frac{1}{(x-y)^2} = \frac{1}{x^2+y^2}$

494. (a) given a+b+c = 0

$= \frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)}$

$+ \frac{1}{(c+a)(a+b)}$

$= \frac{(a+b)+(a+b)+(b+c)}{(a+b)(b+c)(c+a)}$

$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)}$

$= \frac{2 \times 0}{(a+b)(b+c)(c+a)} = 0$

495. (a) $x^2 + y^2 + 2x + 1 = 0$

$(x + 1)^2 + y^2 = 0$

Let $(x + 1)^2 = 0$, $y^2 = 0$

x = – 1, y = 0

Now, $x^{31} + y^{35} = (-1)^{31} + (0)^{35}$

= – 1

496. (c) $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$

$\because x = \frac{1}{y} \Rightarrow xy = 1$

$x + y = \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$

$= \frac{5+1+2\sqrt{5}+5+1-2\sqrt{5}}{5-1}$

$x + y = \frac{12}{4} = 3$

$x + \frac{1}{x} = 3$

$x^2 + \frac{1}{x^2} = (3)^2 - 2 = 7$

Now, $= \frac{x^2+xy+y^2}{x^2-xy+y^2}$

$= \frac{x^2+y^2+xy}{x^2+y^2-xy} = \frac{7+1}{7-1}$

$= \frac{8}{6} = \frac{4}{3}$

497. (b) $\left(x - \frac{1}{x}\right)^2 = 3$

$x - \frac{1}{x} = \sqrt{3}$

Now, $x^2 + \frac{1}{x^2} = (\sqrt{3})^2+2$

$x^2 + \frac{1}{x^2} = 5$

again $(x^2)^3 + \left(\frac{1}{x^2}\right)^3 = (5)^3 - 3 \times 5$

$x^6 + \frac{1}{x^6} = 125 - 15 = 110$

498. (b) $x^4 + 2x^3 + ax^2 + bx + 9$
put $x = 1$
$= 1 + 2 \times 1 + a + b + 9$
$= 1 + 2 + a + b + 9$
$= 13 + a + b$
To make perfect square no., value of a + b must be either 3 or 13
Now, option (b) a = 6, b = 7,
a + b = 12
makes perfect square ($25 = 5^2$)

499. (b) $a^2+b^2+c^2 = x^2+y^2+z^2 = 25$
but b = c = 0 but y = z = 0
a = 4 $x = 5$
Now,
$ax + by + cz = 20$
$4 \times 5 + 0 + 0 = 0$
20 = 20 (satisfy)

Now, $\frac{a+b+c}{x+y+z} = \frac{4+0+0}{5+0+0} = \frac{4}{5}$

500. (c) $\frac{x+a^2+2c^2}{b+c} + \frac{x+b^2+2a^2}{c+a} + \frac{x+c+2b^2}{a+b} = 0$

Let $\frac{x+a^2+2c^2}{b+c} = 0$,

$\frac{x+b^2+2a^2}{c+a} = 0$

$x = -2c^2 - a^2$ ....(i)
$x = -2a^2 - b^2$ ....(ii)

$\frac{x+c+2b^2}{a+b} = 0$, $x = -2b^2 - c$ ..(iii)

From eq (i) (ii) & (iii)
$3x = -2c^2 - a^2 - 2a^2 - b^2 - 2b^2 - c^2$
$3x = -3(a^2 + b^2 + c^2)$
$x = -(a^2 + b^2 + c^2)$

Alternate:

$\frac{x+a^2+2c^2}{b+c} + \frac{x+b^2+2a^2}{c+a} + \frac{x+c^2+2b^2}{a+b} = 0$

put a = b = c = 1

$\frac{x+1+2}{2} + \frac{x+1+2}{2} + \frac{x+1+2}{2} = 0$

$\frac{x+3+x+3+x+3}{2} = 0$

$3x + 9 = 0$
$3x = -9$
$x = -3$
Now, option C
$-[a^2 + b^2 + c^2]$
$= -[1+1+1] = -3$ (satisfy)

501. (a) $a^3 = 117 + b^3$, $a = 3 + b$
$a^3 - b^3$ 117....(i) $a - b = 3$ ...(ii)
put a = 5, b = 2
both eq satisfy
Now, a+b = 5+2 = 7

502. (a) $a + \frac{1}{a} = -2$
but a = –1 (satisfy)
Now,

$a^{1000} + a^{-1000} = a^{1000} + \frac{1}{a^{1000}}$

$= 1^{1000} + \frac{1}{1^{1000}} = 1 + 1 = 2$

503. (a) $x^2 + \frac{1}{x^2} = 98$

$x^2 + \frac{1}{x^2} + 2 = 100$

$\left(x + \frac{1}{x}\right)^2 = 100$

$x + \frac{1}{x} = 10$

$x^3 + \frac{1}{x^3} = 10^3 - 3 \times 10$

$x^3 + \frac{1}{x^3} = 970$

504. (c) $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$
Let a = 2, and b = –1

$2 + \frac{1}{-1} = 1$

1 = 1 True
Now find value of C

$b + \frac{1}{c} = 1$ (Put value of b)

$-1 + \frac{1}{c} = 1$

$\frac{1}{c} = 2 \Rightarrow c = \frac{1}{2}$

So, $c + \frac{1}{a} = \frac{1}{2} + \frac{1}{2} = 1$

505. (b) Given x = y + z
$\therefore$ x – y – z = 0
then $x^3 - y^3 - z^3 = 3xyz$

506. (a)

$\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)} + \frac{1}{(1-d)(1-a)(1-b)}$

$= \frac{1-d+1-a+1-b+1-c}{(1-a)(1-b)(1-c)(1-d)}$

$\frac{4-(a+b+c+d)}{(1-a)(1-b)(1-c)(1-d)}$

$= \frac{4-4}{(1-a)(1-b)(1-c)(1-d)} = 0$

507. (b) $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$
$x^5 - 11x^4 - x^4 + 11x^3 + x^3 - 11x^2 - x^2 + 11x + x - 1$
Put $x = 11$
$= 11^5 - 11.11^4 - 11^4 + 11.11^3 + 11^3 - 11.11^2 - 11^2 + 11.11 + 11 - 1$
$= 11 - 1 = 10$

508. (a) $a = \frac{1}{a-5}$
$a^2 - 5a = 1$
$a^2 - 1 = 5a$ divide by a

$a - \frac{1}{a} = 5$
Square on both side

$a^2 + \frac{1}{a^2} - 2 = 25$

$a^2 + \frac{1}{a^2} = 27$

$a^2 + \frac{1}{a^2} + 2 = 27 + 2$

Adding 2 both side

$$\left(a+\frac{1}{a}\right)^2 = 29$$

$$\left(a+\frac{1}{a}\right) = \sqrt{29}$$

509. (c) $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$

Put, $a = \frac{1}{2}$, $b = 2$, $c = -1$

$\frac{1}{2} + \frac{1}{2} = 2 - 1 = -1 + 2$

$1 = 1 = 1$

$abc = -\frac{1}{2} \times 2x - 1$

$\boxed{abc = -1}$

again put

$a = -\frac{1}{2}$, $b = -2$, $c = 1$

$a + \frac{1}{b} = b + \frac{1}{c} + c + \frac{1}{a}$

$-\frac{1}{2} - \frac{1}{2} = -2 + 1 = 1 - 2$

$-1 = -1 = -1$

equation satistied

$abc = -\frac{1}{2} \times -2 \times 1 = 1$

So abc can be – 1 and + 1.

510. (a) $ax + by = 1$

$bx + ay = \frac{2ab}{a^2+b^2}$

Let, $a = b = 1$

then,

$x + y = 1$ .....(i)

$x + y = \frac{2\times1\times1}{1+1} = 1$ ...(ii)

Now put

$x = y = \frac{1}{2}$ in eq. (i) and (ii)

$\frac{1}{2} + \frac{1}{2} = 1$

and equation (ii)

$\frac{1}{2} + \frac{1}{2} = 1$

Value of $(x^2 + y^2)(a^2 + b^2)$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 (1 + 1)$

$= \left(\frac{1}{4} + \frac{1}{4}\right) \times 2 = \frac{2}{4} \times 2 = 1$

511. (a) $a^3 - 7a - 6$

$(a+1)(a^2-a-6)$

$(a+1)(a+2)(a-3)$

now,

sum of factors

$(a+1)+a+2+a-3=3a$